

Online Appendix for “Production Network Formation, Trade, and Welfare”

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A Proofs and Mathematical Details

For notational convenience, we define $\mathbb{M}_i(\delta) \equiv \int z^\delta dG_i(z)$.

A.1 Proof of Lemma 1

We first note that Problem (10) is a strictly convex optimization problem when $\gamma^B > 1$ and $\gamma^S > 1$. Therefore, there is a unique solution to the problem, and the first order conditions (FOCs) are necessary and sufficient for the solution.

Conjecture that solution entails $n_{id}^B(z) = a_{id}^B z^{\frac{\delta}{\gamma^B}}$, $n_{ui}^S(z) = a_{ui}^S z^{\frac{\delta}{\gamma^S}}$, and $\delta \equiv (\sigma - 1) / \left(1 - \frac{1}{\gamma^B} - \frac{1-\beta}{\gamma^S}\right)$ as in the proposition. By plugging these solutions to (9), we obtain the expression for marginal cost $c_i(z)$ as in equation (12). By plugging these solutions to $r_i^*(z) = \sum_d r_{id}(z) m_{id}^B n_{id}^B(z)$, where $r_{id}(z) \equiv (\tilde{\sigma} \tau_{id} c_i(z))^{1-\sigma} D_d$ is the revenue per buyer link, we obtain the expression for revenue $r_i^*(z)$ as in equation (13).

The FOC with respect to n_{id}^B is given by

$$e_i f_{id}^B (n_{id}^B)^{\gamma^B-1} = \frac{\tilde{\sigma}^{1-\sigma}}{\sigma} m_{id}^B D_d (\tau_{id})^{1-\sigma} \underbrace{\frac{w_i^{\beta(1-\sigma)} (\sum_{u \in \mathcal{N}} n_{ui}^S m_{ui}^S (C_{ui})^{1-\sigma})^{1-\beta}}{z^{1-\sigma}}}_{=c_i(z)^{1-\sigma}} \quad (\text{A.1})$$

The FOC with respect to n_{ui}^S is given by

$$\begin{aligned} e_i f_{ui}^S (n_{ui}^S)^{\gamma^S-1} &= \frac{\tilde{\sigma}^{1-\sigma}}{\sigma} \left\{ \sum_d n_{id}^B m_{id}^B D_d (\tau_{id})^{1-\sigma} \right\} \\ &\times (1-\beta) \frac{w_i^{\beta(1-\sigma)} (\sum_{u' \in \mathcal{N}} n_{u'i}^S m_{u'i}^S (C_{u'i})^{1-\sigma})^{-\beta}}{z^{1-\sigma}} m_{ui}^S (C_{ui})^{1-\sigma} \end{aligned} \quad (\text{A.2})$$

Note that all terms related to z cancel out under the conjectures of $\{n_{id}^B(z), n_{ui}^S(z)\}$ in the proposition. Furthermore, equation (A.1) is rewritten as

$$\begin{aligned} e_i f_{id}^B (a_{id}^B)^{\gamma^B} &= a_{id}^B \frac{\tilde{\sigma}^{1-\sigma}}{\sigma} m_{id}^B D_d (\tau_{id})^{1-\sigma} w_i^{\beta(1-\sigma)} \left(\sum_{u \in \mathcal{N}} a_{ui}^S m_{ui}^S (C_{ui})^{1-\sigma} \right)^{1-\beta} \\ &= a_{id}^B \frac{\tilde{\sigma}^{1-\sigma}}{\sigma} m_{id}^B D_d (\tau_{id})^{1-\sigma} C_i^{1-\sigma} \\ &= a_{id}^B \frac{\tilde{\sigma}^{1-\sigma}}{\sigma} m_{id}^B D_d (\tau_{id})^{1-\sigma} z^{-\frac{\delta}{\gamma^S}(1-\beta)-(\sigma-1)} c_i(z)^{1-\sigma} \\ &= \frac{1}{\sigma} r_{id}(z) m_{id}^B a_{id}^B z^{-\frac{\delta}{\gamma^S}(1-\beta)-(\sigma-1)} \\ &= \frac{1}{\sigma} r_{id}(z) m_{id}^B n_{id}^B(z) z^{-\frac{\delta}{\gamma^B}-\frac{\delta}{\gamma^S}(1-\beta)-(\sigma-1)} \\ \iff e_i f_{id}^B (a_{id}^B)^{\gamma^B} z^\delta &= \frac{1}{\sigma} r_{id}(z) m_{id}^B n_{id}^B(z). \end{aligned} \quad (\text{A.3})$$

By integrating wrt z using density function $G_i(\cdot)$ and multiplying by N_i ,

$$e_i f_{id}^B (a_{id}^B)^{\gamma^B} N_i \mathbb{M}_i(\delta) = \frac{1}{\sigma} X_{id} \iff a_{id}^B = \left(\Gamma_i^B \frac{X_{id}}{e_i f_{id}^B} \right)^{\frac{1}{\gamma^B}}, \quad (\text{A.4})$$

where we define $\Gamma_i^B = \frac{1}{\sigma \mathbb{M}_i(\delta) N_i}$, and $X_{id} = N_i \int r_{id}(z) m_{id}^B n_{id}^B(z) dG_i(z)$ is the total intermediate goods revenue sold by firms in i to firms in d . This expression corresponds to the one in Lemma 1.

Equation (A.2) is rewritten as

$$\begin{aligned} e_i f_{ui}^S (a_{ui}^S)^{\gamma^S-1} &= \frac{\tilde{\sigma}^{1-\sigma}}{\sigma} \underbrace{\left\{ \sum_d a_{id}^B m_{id}^B D_d \tau_{id}^{1-\sigma} \right\}}_{=D_i^*=z^{-\delta} r_i^*(z) (\tilde{\sigma} C_i)^{\sigma-1}} (1-\beta) \\ &\quad \times \underbrace{w_i^{\beta(1-\sigma)} \left(\sum_{u'} a_{u'i}^S m_{u'i}^S (C_{u'i})^{1-\sigma} \right)^{-\beta} m_{ui}^S (C_{ui})^{1-\sigma}}_{=C_i^{1-\sigma} (a_{ui}^S)^{-1} \Lambda_{ui}} \\ \iff e_i f_{ui}^S (a_{ui}^S)^{\gamma^S} z^\delta &= \frac{1}{\sigma} (1-\beta) \Lambda_{ui} r_i^*(z) \end{aligned} \quad (\text{A.5})$$

where $\Lambda_{ui} = X_{ui} / \sum_{u'} X_{u'i}$. By integrating wrt z using density function $G_i(\cdot)$ and multiplying by N_i ,

$$\begin{aligned} e_i f_{ui}^S (a_{ui}^S)^{\gamma^S} N_i \mathbb{M}_i(\delta) &= \frac{1}{\sigma} (1-\beta) \Lambda_{ui} R_i \\ \iff a_{ui}^S &= \left(\frac{1}{\sigma \mathbb{M}_i(\delta) N_i} \frac{(1-\beta) R_i X_{ui}}{e_i f_{ui}^S E_i} \right)^{\frac{1}{\gamma^S}} = \left(\Gamma_i^S \frac{X_{ui}}{e_i f_{ui}^S} \right)^{\frac{1}{\gamma^S}}, \end{aligned} \quad (\text{A.6})$$

where $R_i = \sum_d X_{id}$ is intermediate goods sales by firms in i (excluding sales to retailers), $E_i = \sum_u X_{ui}$ is intermediate goods expenditure by firms in i , and $\Gamma_i^S \equiv \frac{1-\beta}{\sigma \mathbb{M}_i(\delta) N_i} \frac{R_i}{E_i}$. Note that, under trade balance, $R_i = E_i$. This expression corresponds to the one in Lemma 1.

Finally, firms' profit subtracting the expenditure for relationship formation activity is given by

$$\tilde{\pi}_i(z) = \frac{1}{\sigma} r_i^*(z) - e_i \left\{ \sum_{d \in \mathcal{N}} f_{id}^B \frac{(n_{id}^B)^{\gamma^B}}{\gamma^B} + \sum_{u \in \mathcal{N}} f_{ui}^S \frac{(n_{ui}^S)^{\gamma^S}}{\gamma^S} \right\} = \frac{1}{\sigma} \left\{ 1 - \frac{1}{\gamma^B} - \frac{1-\beta}{\gamma^S} \right\} r_i^*(z),$$

where we used equations (A.3) and (A.5). This equation coincides with equation (14).

A.2 Proof of Lemma 2

We first obtain the intensive margin of trade flows, i.e., the average transaction volume per relationship (intensive margin). Noting that the average sales per buyer by firms in u with produc-

tivity z selling in d is given by $r_{ud}(z) = (\tilde{\sigma}\tau_{ud}c_u(z))^{1-\sigma} D_d$,

$$\bar{r}_{ud} = \frac{\int r_{ud}(z) n_{ud}^B(z) dG_u(z)}{\int n_{ud}^B(z) dG_u(z)} = (\tilde{\sigma}\tau_{ud})^{1-\sigma} D_d \frac{\int c_u(z)^{1-\sigma} n_{ud}^B(z) dG_u(z)}{\int n_{ud}^B(z) dG_u(z)}.$$

Using the expression for the marginal costs and buyer relationship-formation effort from Lemma 1,

$$\bar{r}_{ud} = \varrho^I (\tau_{ud})^{1-\sigma} \left(\frac{\mathbb{M}_u(\delta)}{\mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right)} C_u^{1-\sigma} \right) D_d, \quad (\text{A.7})$$

with $\varrho^I = \tilde{\sigma}^{1-\sigma}$, which corresponds to $\bar{r}_{ud}$ in equation (18) with $\zeta_u^I = \frac{\mathbb{M}_u(\delta)}{\mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right)} C_u^{1-\sigma}$ and $\xi_d^I = D_d$.

We next obtain the extensive margin, i.e., the measure of supplier-to-buyer relationships from supplier location u to buyer location d , M_{ud} . From equation (16),

$$M_{ud} = \kappa_{ud} \left(N_d a_{ud}^S \mathbb{M}_d \left(\frac{\delta}{\gamma^S} \right) \right)^{\lambda^S} \left(N_u a_{ud}^B \mathbb{M}_u \left(\frac{\delta}{\gamma^B} \right) \right)^{\lambda^B} = \kappa_{ud}^* \frac{X_{ud}^{\tilde{\lambda}^S + \tilde{\lambda}^B}}{e_u^{\tilde{\lambda}^B} e_d^{\tilde{\lambda}^S}}$$

where $\kappa_{ud}^* = \kappa_{ud} (f_{ud}^S)^{-\tilde{\lambda}^S} (f_{ud}^B)^{-\tilde{\lambda}^B} \left(N_d \mathbb{M}_d \left(\frac{\delta}{\gamma^S} \right) \right)^{\lambda^S} \left(N_u \mathbb{M}_u \left(\frac{\delta}{\gamma^B} \right) \right)^{\lambda^B} (\Gamma_d^S)^{\tilde{\lambda}^S} (\Gamma_u^B)^{\tilde{\lambda}^B}$. Plugging $X_{ud} = \bar{r}_{ud} M_{ud}$ into this equation yields

$$\begin{aligned} M_{ud} &= \kappa_{ud}^* (M_{ud} \bar{r}_{ud})^{\tilde{\lambda}^S + \tilde{\lambda}^B} e_u^{-\tilde{\lambda}^B} e_d^{-\tilde{\lambda}^S} \\ &= (\kappa_{ud}^*)^{\frac{1}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} \bar{r}_{ud}^{\frac{\tilde{\lambda}^S + \tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} e_u^{-\frac{\tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} e_d^{-\frac{\tilde{\lambda}^S}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} \\ &= (\kappa_{ud}^*)^{\frac{1}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} \left[\tau_{ud}^{1-\sigma} \varrho^I C_u^{1-\sigma} \frac{\mathbb{M}_u(\delta)}{\mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right)} D_d \right]^{\frac{\tilde{\lambda}^S + \tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} e_u^{-\frac{\tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} e_d^{-\frac{\tilde{\lambda}^S}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}}, \end{aligned} \quad (\text{A.8})$$

which corresponds to M_{ud} in equation (18) with $\rho^E = (\varrho^I)^{\frac{\tilde{\lambda}^S + \tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}}$, and

$$\chi_{ud}^E = \left[\kappa_{ud} (f_{ud}^B)^{-\tilde{\lambda}^B} (f_{ud}^S)^{-\tilde{\lambda}^S} (\tau_{ud}^{1-\sigma})^{\tilde{\lambda}^B + \tilde{\lambda}^S} \right]^{\frac{1}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}}, \quad (\text{A.9})$$

$$\zeta_u^E = \left(\frac{\mathbb{M}_u(\delta)}{\mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right)} \right)^{\frac{\tilde{\lambda}^S + \tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} C_u^{\frac{\tilde{\lambda}^S + \tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B} (1-\sigma)} e_u^{-\frac{\tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} \left(N_u \mathbb{M}_u \left(\frac{\delta}{\gamma^B} \right) \right)^{\lambda^B} (\Gamma_u^B)^{\tilde{\lambda}^B}, \quad (\text{A.10})$$

$$\xi_d^E = D_d^{\frac{\tilde{\lambda}^S + \tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} e_d^{-\frac{\tilde{\lambda}^S}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} \left(N_d \mathbb{M}_d \left(\frac{\delta}{\gamma^S} \right) \right)^{\lambda^S} (\Gamma_d^S)^{\tilde{\lambda}^S} \quad (\text{A.11})$$

A.3 Market Clearing Conditions

Intermediate goods for each firm ω are used as inputs for production by other firms and local retailers:

$$q_i^R(\omega) + \sum_{d \in \mathcal{N}} \int_{\psi: \omega \in S_{id}(\psi)} q_{id}(\omega, \psi) \tau_{id} d\psi = q_i(\omega). \quad (\text{A.12})$$

Relationship-formation efforts use local labor and retail goods:

$$\int_{\omega \in \Omega_i(\omega)} \left(\sum_d n_{id}^B(z) + \sum_u n_{ui}^S(\omega) \right) d\omega = \left(\frac{L_i^A}{\mu} \right)^\mu \left(\frac{Q_i^A}{1-\mu} \right)^{1-\mu}, \quad (\text{A.13})$$

where L_i^A and Q_i^A correspond to the aggregate amount of labor and retail goods used for relationship-formation activity.

Retail goods are consumed by final consumers and used for relationship-formation efforts:

$$L_i Q_i^F + Q_i^A = Q_i. \quad (\text{A.14})$$

Labor is used for intermediate goods production and for relationship-formation efforts:

$$\int_{\omega \in \Omega_i(\omega)} l_i(\omega) d\omega + L_i^A = L_i. \quad (\text{A.15})$$

A.4 Proof of Proposition 1

Preliminaries We first obtain several useful expressions. First, we derive $C_{ui}^{1-\sigma} \equiv \int (\tilde{\sigma} C_u(z) \tau_{ui})^{1-\sigma} dG_{ui}^B(z)$, where $G_{ui}^B(\cdot)$ denotes the weighted distribution of z by buyer relationship-formation intensity, given by

$$dG_{ui}^B(z) = \frac{n_{ui}^B(z) dG_u(z)}{\int n_{ui}^B(z') dG_u(z')} = \frac{z^{\frac{\delta}{\gamma^B}} dG_u(z)}{\int z'^{\frac{\delta}{\gamma^B}} dG_u(z')} = \frac{z^{\frac{\delta}{\gamma^B}} dG_u(z)}{\mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right)}.$$

Therefore we have

$$C_{ui}^{1-\sigma} = \int \left(\tilde{\sigma} C_u z^{-\frac{\delta}{\gamma^S} \frac{1-\beta}{\sigma-1} - 1} \tau_{ui} \right)^{1-\sigma} dG_{ui}^B(z) = (\tilde{\sigma} C_u \tau_{ui})^{1-\sigma} \frac{\mathbb{M}_u(\delta)}{\mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right)}, \quad (\text{A.16})$$

where we used the fact that $\delta = \left(-\frac{\delta}{\gamma^S} \frac{1-\beta}{\sigma-1} - 1 \right) (1-\sigma) + \frac{\delta}{\gamma^B}$.

Next, we derive D_d . Notice the aggregate intermediate input expenditure E_d satisfies

$$\begin{aligned}
E_d &= \sum_u N_u \int (\tau_{ud})^{1-\sigma} D_d (\tilde{\sigma} c_u(z))^{1-\sigma} n_{ud}^B(z) m_{ud}^B dG_u(z) \\
&= D_d \sum_u N_u (\tau_{ud})^{1-\sigma} (\tilde{\sigma})^{1-\sigma} \int (C_u)^{1-\sigma} a_{ud}^B m_{ud}^B z^{\frac{\delta}{\gamma^B} + \frac{\delta}{\gamma^S}(1-\beta) + (\sigma-1)} dG_u(z) \\
&= D_d \sum_u (\tau_{ud})^{1-\sigma} (\tilde{\sigma})^{1-\sigma} (C_u)^{1-\sigma} N_u a_{ud}^B m_{ud}^B \mathbb{M}_u(\delta) \\
&= D_d \sum_u (\tau_{ud})^{1-\sigma} (\tilde{\sigma})^{1-\sigma} (C_u)^{1-\sigma} N_d a_{ud}^S m_{ud}^S \frac{\mathbb{M}_d\left(\frac{\delta}{\gamma^S}\right)}{\mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right)} \mathbb{M}_u(\delta) \quad (\text{from 17 and } \overline{M}_{ud}^S, \overline{M}_{ud}^B) \\
&= D_d N_d \mathbb{M}_d\left(\frac{\delta}{\gamma^S}\right) \sum_u C_{ud}^{1-\sigma} a_{ud}^S m_{ud}^S \quad (\text{from equation A.16}) \\
&= D_d N_d \mathbb{M}_d\left(\frac{\delta}{\gamma^S}\right) \left[w_d^{-\beta} C_d\right]^{\frac{1-\sigma}{1-\beta}} \quad (\text{from equation 12})
\end{aligned}$$

Denote nominal labor compensation to intermediate input expenditure ratio:

$$\varsigma^L \equiv \frac{w_d L_d}{E_d} = \frac{\frac{\sigma-1}{\sigma} \beta + \frac{1}{\sigma} \mu \left(\frac{1}{\gamma^B} + \frac{1-\beta}{\gamma^S} \right)}{\frac{\sigma-1}{\sigma} (1-\beta)}. \quad (\text{A.17})$$

Then,

$$D_d = \frac{1}{\varsigma^L} \frac{L_d}{N_d \mathbb{M}_d\left(\frac{\delta}{\gamma^S}\right)} (w_d)^{\frac{1-\beta\sigma}{1-\beta}} (C_d)^{\frac{\sigma-1}{1-\beta}}. \quad (\text{A.18})$$

Finally, from equation (18) and Appendix A.2, total margin trade flows are given by

$$X_{ud} = \varrho \chi_{ud} \zeta_u \xi_d \quad (\text{A.19})$$

where $\rho = \varrho^I \varrho^E$, $\chi_{ud} = \chi_{ud}^I \chi_{ud}^E$, and

$$\begin{aligned}
\zeta_u &= C_u^{-\varepsilon} e_u^{-\frac{\tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} \tilde{K}_u^U, \quad \tilde{K}_u^U \equiv \left(\frac{\mathbb{M}_u(\delta)}{\mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right)} \right)^{\frac{1}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} \left(N_u \mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right) \right)^{\lambda^B} (\Gamma_u^B)^{\tilde{\lambda}^B} \\
\xi_d &= w_d^{\frac{1-\beta\sigma}{1-\beta} \frac{\varepsilon}{\sigma-1}} C_d^{\frac{\varepsilon}{1-\beta}} e_d^{-\frac{\tilde{\lambda}^S}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} K_d^D, \quad K_d^D \equiv \left(N_d \mathbb{M}_d\left(\frac{\delta}{\gamma^S}\right) \right)^{\lambda^S} (\Gamma_d^S)^{\tilde{\lambda}^S}
\end{aligned}$$

Part (a) From the accounting identity,

$$w_i L_i = \varsigma^L \sum_d X_{id}, \quad (\text{A.20})$$

where ς^L is the nominal labor compensation to intermediate input expenditure ratio as defined in equation (A.17). Plugging equation (A.19) into this equation, and after some algebra, we obtain equation (20) with $K_i^U \equiv \varrho \varsigma^L \tilde{K}_i^U$.

Next, by combining equation (A.20) and the trade balancing condition (implied by the market clearing conditions,¹ we obtain:

$$w_i L_i = \varsigma^L \sum_u X_{ui}. \quad (\text{A.21})$$

Plugging equation (A.19) into this equation, we obtain equation (21).

Part (b) We apply Allen et al. (2024). Define the matrices

$$\Gamma \equiv \begin{bmatrix} 1 + c_1 & c_2 \\ 1 - \delta_G & -c_3 \end{bmatrix}, \quad B \equiv \begin{bmatrix} \delta_G & c_3 \\ -c_1 & -c_2 \end{bmatrix}$$

where we define

$$\delta_G = \left(\frac{1 - \beta\sigma}{1 - \beta} - \tilde{\lambda}^S \mu \right) \frac{\varepsilon}{\sigma - 1}$$

$$c_1 = \tilde{\lambda}^B \frac{\varepsilon}{\sigma - 1} \mu, \quad c_2 = \varepsilon \left(1 + \tilde{\lambda}^B \frac{1 - \mu}{\sigma - 1} \right), \quad c_3 = \varepsilon \left(\frac{1}{1 - \beta} - \tilde{\lambda}^S \frac{1 - \mu}{\sigma - 1} \right)$$

where $c_1 > 0$ and $c_2 > 0$ under our model parameter assumptions. A sufficient condition for the equilibrium uniqueness is that the spectral radius of $A = |B\Gamma^{-1}|$ is equal to 1, where

$$\begin{aligned} B\Gamma^{-1} &= \frac{1}{-c_3(1 + c_1) - c_2(1 - \delta_G)} \begin{bmatrix} \delta_G & c_3 \\ -c_1 & -c_2 \end{bmatrix} \begin{bmatrix} -c_3 & -c_2 \\ -(1 - \delta_G) & 1 + c_1 \end{bmatrix} \\ &= \frac{1}{-c_3(1 + c_1) - c_2(1 - \delta_G)} \begin{bmatrix} -c_3 & -\delta_G c_2 + c_3(1 + c_1) \\ c_1 c_3 + (1 - \delta_G) c_2 & -c_2 \end{bmatrix} \end{aligned}$$

We now show that, when $\delta_G \leq 1$ and $\frac{\beta(\sigma-1)}{1-\beta} \geq (1 - \mu) (\tilde{\lambda}^B + \tilde{\lambda}^S)$ as assumed in the proposition, the largest eigenvalue of $|B\Gamma^{-1}|$ is indeed less than one. From the second condition, we have $c_3 > c_2 > 0$. Furthermore, together with $\delta_G \leq 1$, $-\delta_G c_2 + c_3(1 + c_1) \geq c_1 c_3 + (1 - \delta_G) c_2 > 0$. Therefore,

$$|B\Gamma^{-1}| = \frac{1}{c_3(1 + c_1) + c_2(1 - \delta_G)} \begin{bmatrix} c_3 & -\delta_G c_2 + c_3(1 + c_1) \\ c_1 c_3 + (1 - \delta_G) c_2 & c_2 \end{bmatrix}$$

Note that the sum of the rows for the first column and second column are both one. Therefore, from Collatz–Wielandt Formula (see Allen et al. (2024)), the largest eigenvalue of $|B\Gamma^{-1}|$ is one under this condition. Therefore, when $\delta_G \leq 1$ and $\frac{\beta(\sigma-1)}{1-\beta} \geq (1 - \mu) (\tilde{\lambda}^B + \tilde{\lambda}^S)$, the equilibrium exists and it is unique up to scale.

¹By integrating intermediate goods clearing condition (A.12) across firms, we have $P_i Q_i + \sum_d X_{id} = \tilde{R}_i$, where $\tilde{R}_i$ is firms' aggregate revenue. From the accounting identity, $\tilde{R}_i = \sum_u X_{ui} + \Pi_i + w_i L_i + P_i Q_i^A$. The retail goods market clearing (A.14) implies that $\Pi_i + w_i L_i + P_i Q_i^A = P_i Q_i$. Putting together, we have $\sum_u X_{ui} = \sum_d X_{id}$.

A.5 Exact-hat Algebra

Consider the changes in fundamentals $\{\hat{\tau}_{ud}, \hat{\kappa}_{ud}, \hat{f}_{ud}^B, \hat{f}_{ud}^S\}$, where $\hat{x} = x'/x$ denote the proportional change in variable x and x' denotes the value of x after the change in fundamentals. The changes in equilibrium wages $\{\hat{w}_i\}$ and cost shifters $\{\hat{C}_i\}$ are given by the solution to the following set of equations:

$$\hat{w}_i^{1+\bar{\lambda}^B \mu \frac{\varepsilon}{\sigma-1}} \hat{C}_i^{\varepsilon(1+\bar{\lambda}^B \frac{1-\mu}{\sigma-1})} = \sum_d \Psi_{id} \hat{\chi}_{id} \hat{w}_d^{\left(\frac{1-\beta\sigma}{1-\beta} - \bar{\lambda}^S \mu\right) \frac{\varepsilon}{\sigma-1}} \hat{C}_d^{\varepsilon\left(\frac{1}{1-\beta} - \bar{\lambda}^S \frac{1-\mu}{\sigma-1}\right)}, \quad (\text{A.22})$$

$$\hat{w}_i^{1-\left(\frac{1-\beta\sigma}{1-\beta} - \bar{\lambda}^S \mu\right) \frac{\varepsilon}{\sigma-1}} \hat{C}_i^{-\varepsilon\left(\frac{1}{1-\beta} - \bar{\lambda}^S \frac{1-\mu}{\sigma-1}\right)} = \sum_u \Lambda_{ui} \hat{\chi}_{ui} \hat{w}_u^{-\bar{\lambda}^B \mu \frac{\varepsilon}{\sigma-1}} \hat{C}_u^{-\varepsilon\left(1+\bar{\lambda}^B \frac{1-\mu}{\sigma-1}\right)}, \quad (\text{A.23})$$

where $\Psi_{id} = X_{id}/\sum_n X_{in}$ and $\Lambda_{ui} = X_{ui}/\sum_n X_{ni}$ are import and export shares in the baseline equilibrium, and

$$\hat{\chi}_{ud} = \hat{\tau}_{ud}^{\varepsilon} \left[\hat{\kappa}_{ud} \left(\hat{f}_{ud}^B \right)^{-\bar{\lambda}^B} \left(\hat{f}_{ud}^S \right)^{-\bar{\lambda}^S} \right]^{\frac{1}{1-\bar{\lambda}^S - \bar{\lambda}^B}}. \quad (\text{A.24})$$

A.6 Planning Problem and Proof of Proposition 2 and Corollary 1

Formal Definition of Planning Problem Given the set of subsidies/taxes described in Section 3, the government budget constraint is given by

$$\sum_i T_i^F L_i + \sum_{i,d} \frac{s_{id}^I}{1-s_{id}^I} X_{id} + \sum_{i,d} \frac{t_{id}^B}{1+t_{id}^B} \frac{1}{\sigma} \frac{1}{\gamma^B} X_{id} + \sum_{i,d} \frac{t_{id}^S}{1+t_{id}^S} \frac{1}{\sigma} \frac{1-\beta}{\gamma^S} X_{id} = 0, \quad (\text{A.25})$$

where $\frac{1}{\sigma} \frac{1}{\gamma^B} X_{id}$ and $\frac{1}{\sigma} \frac{1-\beta}{\gamma^S} X_{id}$ correspond to the aggregate expenditure for buyer and supplier relationship-formation effort, respectively. Then, defining the equilibrium prices and quantities by

$$\mathcal{X} \equiv \{X_{id}, p_{id}(v, \omega), P_i, e_i, w_i, C_i, q_{id}(\omega, \psi), q_i^R(\omega), Q_i^F, Q_i^A, n_{id}^B(\omega), n_{ui}^S(\omega), l_i(\omega), L_i^A\},$$

the planning problem is defined by

$$\begin{aligned} & \max_{\{\{s_{id}^I, t_{id}^B, t_{id}^S\}, T_i^F\}, \mathcal{X}} \sum_i \psi_i L_i Q_i^F \\ & \text{s.t.} \quad P_i Q_i^F = w_i + \frac{\Pi_i}{L_i} + T_i^F \\ & \quad X_{id} = \int_{\omega \in \Omega_i, \psi \in \Omega_d} q_{id}(\omega, \psi) dG_i(\omega) dG_d(\psi) \\ & \quad (1)-(3), (5)-(6), (9)-(17), (22), (23), (A.12)-(A.15), (A.25) \end{aligned} \quad (\text{A.26})$$

Proof of Proposition 2 We proceed in two steps. First, we solve for a relaxed planning problem (A.26) where the planner directly specifies the aggregate allocation. Next, we show that the planner can implement this allocation using the subsidies/taxes available to the planner.

Consider the following relaxed planning problem:

$$\begin{aligned}
& \max_{\{Q_i^F, Q_i^A, L_i^A, L_i^P, A_{ui}^S, A_{ui}^B, q_i^R, q_{id}, M_{ui}\}} \sum_i \psi_i L_i Q_i^F & (A.27) \\
& \text{s.t.} \quad Q_i^A + L_i Q_i^F = g_i(q_i^R) & [P_i] \\
& \quad L_i^A + L_i^P = L_i & [w_i] \\
& \quad \sum_u A_{ui}^S + \sum_d A_{id}^B = \left(\frac{L_i^A}{\mu}\right)^\mu \left(\frac{Q_i^A}{1-\mu}\right)^{1-\mu} & [e_i] \\
& \quad q_i^R + \sum_d q_{id} \tau_{id} = Z_i \left(\frac{L_i^P}{\beta}\right)^\beta \left(\frac{\left(\sum_u M_{ui}^{\frac{1}{\sigma}} q_{ui}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}{1-\beta}\right)^{1-\beta} & [C_i] \\
& \quad M_{ui} = \kappa_{ui} \left(\frac{\gamma^B}{f_{ui}^B} A_{ui}^B\right)^{\frac{\lambda^B}{\gamma^B}} \left(\frac{\gamma^S}{f_{ui}^S} A_{ui}^S\right)^{\frac{\lambda^S}{\gamma^S}} & [\eta_{ui}]
\end{aligned}$$

where A_{ui}^B and A_{ui}^S correspond to the aggregate buyer and supplier relationship-formation effort between suppliers in u and buyers in i , and L_i^P is the aggregate labor used for production. The brackets in each constraint represent the Lagrange multipliers. In the final constraint, we combined the matching technology (16) and relationship-formation effort (8). In the second to last constraint, we define q_i^R and q_{ui} as aggregate intermediate good quantify (not per firm or link).

Notice that we dropped the firm identifiers ω within each location, because the planner has no policy tool to discriminate firms within a location. We later verify that these allocations are achieved given tax instruments $\{\{s_{id}^I, t_{id}^B, t_{id}^S\}, T_i\}$. The FOCs of this problem with respect to $Q_i^F, Q_i^A, q_i^R, L_i^A, L_i^P, A_{ui}^B, A_{ui}^S, M_{ui}, q_{ui}$ are given by:

$$P_i = \psi_i \quad (A.28)$$

$$P_i = e_i A_i (1-\mu) \frac{1}{Q_i^A} \quad (A.29)$$

$$C_i = P_i g_i'(q_i^R) \quad (A.30)$$

$$w_i = e_i A_i \mu \frac{1}{L_i^A} \quad (A.31)$$

$$w_i = \beta C_i q_i \frac{1}{L_i^P} \quad (A.32)$$

$$e_u = \eta_{ui} \frac{\lambda^B}{\gamma^B} M_{ui} \frac{1}{A_{ui}^B} \quad (A.33)$$

$$e_i = \eta_{ui} \frac{\lambda^S}{\gamma^S} M_{ui} \frac{1}{A_{ui}^S} \quad (A.34)$$

$$\eta_{ui} = \frac{1}{\sigma - 1} (1 - \beta) C_i q_i \Lambda_{ui} \frac{1}{M_{ui}} \quad (\text{A.35})$$

$$\tau_{ui} C_u = (1 - \beta) C_i q_i \Lambda_{ui} \frac{1}{q_{ui}} \quad (\text{A.36})$$

where $A_i \equiv \sum_u A_{ui}^S + \sum_d A_{id}^B$, $q_i \equiv q_i^R + \sum_d q_{id} \tau_{id}$, and $\Lambda_{ui} \equiv \frac{M_{ui}^{\frac{1}{\sigma}} q_{ui}^{\frac{\sigma-1}{\sigma}}}{\sum_\ell M_{\ell i}^{\frac{1}{\sigma}} q_{\ell i}^{\frac{\sigma-1}{\sigma}}}$. From (A.33), (A.34), (A.35),

$$\frac{\lambda^B}{\gamma^B} \frac{1}{\sigma - 1} (1 - \beta) C_i q_i \Lambda_{ui} = e_u A_{ui}^B \quad (\text{A.37})$$

$$\frac{\lambda^S}{\gamma^S} \frac{1}{\sigma - 1} (1 - \beta) C_i q_i \Lambda_{ui} = e_i A_{ui}^S \quad (\text{A.38})$$

We now consider the equilibrium subsidies/taxes that implement this allocation. Suppose that the Lagrange multipliers $\{P_i, w_i, e_i, C_i\}$ correspond to equilibrium prices. Then, from firms' profit maximization given sales subsidy,

$$(1 - s_{ui}^I) \frac{\sigma}{\sigma - 1} \tau_{ui} C_u q_{ui} = (1 - \beta) C_i q_i \Lambda_{ui} \quad (\text{A.39})$$

By comparing this condition with (A.36), the optimal s_{ui}^I satisfies $1 - s_{ui}^I = 1 - s^I = \frac{\sigma-1}{\sigma} \iff s^I = 1/\sigma$.

Next, following Lemma 1 applied to the economy with taxes, equilibrium allocation for buyer and supplier relationship-formation effort is given by

$$(1 + t_{ui}^B) e_u A_{ui}^B = \frac{1}{\sigma} \frac{1}{\gamma^B} X_{ui} \frac{1}{1 - s^I} \quad (\text{A.40})$$

$$(1 + t_{ui}^S) e_i A_{ui}^S = \frac{1}{\sigma} \frac{1 - \beta}{\gamma^S} X_{ui} \frac{R_i}{E_i} \frac{1}{1 - s^I} \quad (\text{A.41})$$

where we accommodate the possibility that $R_i \neq E_i$ due to the potential trade imbalance from taxes and transfers. By comparing these equations with (A.37) and (A.38), and by noting that $X_{ui} = (1 - \beta) C_i q_i \Lambda_{ui}$, we obtain the expressions in Proposition 2.

Finally, we confirm that these allocations are indeed consistent with the equilibrium conditions. The constraints in (A.27) coincide with resource constraints, production of relationship-formation effort, and matching technology; (A.32) and (A.36) coincide with intermediate goods producers' optimal production decisions; (A.30) coincides with retailers' optimal production decisions; (A.29), (A.31), (A.40) and (A.41) coincide with firms' optimal relationship-formation decisions. Finally, assume that lump-sum transfers are set such that

$$T_i^F = P_i Q_i^F - \frac{\Pi_i^F}{L_i} - w_i$$

By inserting these equilibrium allocations to equation (A.25), we can verify that these allocations satisfy the government budget constraints from market clearing conditions.

Proof of Corollary 1 We first prove that Condition (26) is a necessary condition for the constrained efficiency (i.e., “if” statement). Suppose Condition (26) holds. Suppose that the taxes/subsidies are set according to Corollary 1, i.e., $s_{id}^I = 1/\sigma$ and $t_{id}^B = t_{id}^S = 0$ for all i, d . Suppose also that all supplier subsidies are financed by the local lump-sum taxes, i.e.,

$$T_i^F = - \sum_d \frac{s_{id}^I}{1 - s_{id}^I} X_{id} = - \frac{1}{\sigma - 1} \sum_d X_{id}$$

Clearly these taxes and transfers satisfy the government budget constraint (A.25) and all equilibrium conditions. Together with Proposition 2, these taxes/subsidies are indeed optimal. Furthermore, from equation (A.28), ψ_i is proportional to equilibrium retail prices P_i . Finally, since the taxes are financed within each location, trade balance holds from market clearing conditions, i.e., $E_i = R_i$ for all i . This completes the proof of the “if” statement.

Next, we prove that Condition (26) is a sufficient condition for the constrained efficiency (i.e., “only if” statement). From Proposition 2, $\lambda^B \neq 1$ cannot be optimal under the taxes/subsidies in according to Corollary 1. Furthermore, $\lambda^S \neq (1 - \beta) \frac{E_d}{R_d}$ cannot be optimal under these taxes/subsidies. At the same time, the only case where $\lambda^S = (1 - \beta) \frac{E_d}{R_d}$ holds for all d is when trade balance holds, i.e., $E_d = R_d$, and $\lambda^S = 1 - \beta$. Hence, $\lambda^S \neq 1 - \beta$ cannot be optimal under the taxes/subsidies in Corollary 1. Therefore, $\lambda^B \neq 1$ or $\lambda^S \neq 1 - \beta$ cannot attain constrained efficiency. This completes the proof of the “only if” statement.

A.7 Proof of Proposition 3

Assume that the welfare weights coincide with the retail goods prices, $\psi_i = P_i$. Denote the nominal GDP by Y_i^F . Then, by noting that we define global nominal GDP as the numeraire ($\sum_i Y_i^F = 1$), and that $d \ln C_i = d \ln P_i$ from cost minimization of retailers (equation 6),

$$d \ln \mathcal{W} = \sum_i Y_i^F (d \ln Y_i^F - d \ln C_i) = - \sum_i Y_i^F d \ln C_i. \quad (\text{A.42})$$

We first characterize $d \ln C_i$ through forward (cost) linkages. By applying Shephard’s Lemma to firm’s cost minimization problem (12),

$$d \ln C_i = \beta d \ln w_i + (1 - \beta) \sum_u \Lambda_{ui} \left(d \ln C_u + d \ln \tau_{ui} - \frac{1}{\sigma - 1} d \ln M_{ui} \right). \quad (\text{A.43})$$

Furthermore, from Lemmas 1 and 2,

$$\begin{aligned} d \ln M_{ui} = & \left(\tilde{\lambda}^S + \tilde{\lambda}^B \right) (d \ln \Lambda_{ui} + d \ln w_i) \\ & - \tilde{\lambda}^S (\mu d \ln w_u + (1 - \mu) d \ln C_u) - \tilde{\lambda}^B (\mu d \ln w_i + (1 - \mu) d \ln C_i), \end{aligned} \quad (\text{A.44})$$

where we used the fact that $d \ln X_{ui} = d \ln \Lambda_{ui} + d \ln E_i = d \ln \Lambda_{ui} + d \ln w_i$ and $d \ln e_i =$

$\mu d \ln w_i + (1 - \mu) d \ln C_i$. Reformulating,

$$\left(1 - (1 - \beta) \frac{1 - \mu}{\sigma - 1} \tilde{\lambda}^B\right) d \ln C_i = (1 - \beta) \sum_u \Lambda_{ui} \left(\left(1 + \frac{1 - \mu}{\sigma - 1} \tilde{\lambda}^S\right) d \ln C_u + d \ln \tau_{ui} + d \ln Z_{ui} \right),$$

where

$$d \ln Z_{ui} = \frac{1}{\sigma - 1} \left(\tilde{\lambda}^S \mu d \ln w_u - \left(\tilde{\lambda}^S + \tilde{\lambda}^B (1 - \mu) - \frac{\beta}{1 - \beta} (\sigma - 1) \right) d \ln w_i \right)$$

In matrix notation,

$$d \ln \mathbf{C} = \Phi^{-1} (1 - \beta) [\mathbf{\Lambda} \cdot (\boldsymbol{\tau} + \mathbf{Z})]' \mathbf{1}, \quad (\text{A.45})$$

where $d \ln \mathbf{C}$ is a $1 \times |\mathcal{N}|$ vector of $\{d \ln C_i\}$, $\mathbf{\Lambda}$, $\boldsymbol{\tau}$, $\mathbf{Z}$ are matrices of $\{\Lambda_{ui}\}$, $\{d \ln \tau_{ui}\}$, and $\{d \ln Z_{ui}\}$, $\mathbf{1}$ is a $1 \times |\mathcal{N}|$ vector of ones, and

$$\Phi = \left[\left(1 - \underbrace{(1 - \beta) \frac{1 - \mu}{\sigma - 1} \tilde{\lambda}^B}_{\equiv c^B} \right) \mathbf{I} - (1 - \beta) \left(1 + \underbrace{\frac{1 - \mu}{\sigma - 1} \tilde{\lambda}^S}_{\equiv c^S} \right) \mathbf{\Lambda} \right],$$

where $\mathbf{I}$ is the identity matrix. Together,

$$d \ln \mathcal{W} = -\mathbf{Y}^{\mathbf{F}'} \Phi^{-1} (1 - \beta) [\mathbf{\Lambda} \cdot (\boldsymbol{\tau} + \mathbf{Z})]' \mathbf{1}. \quad (\text{A.46})$$

We next characterize nominal GDP Y_i^F through backward (demand) linkages. Let us define $\tilde{R}_i$ as location i 's final consumption expenditure (Y_i^F) plus the intermediate goods sales to other firms. Then, $\tilde{R}_i = \frac{1}{\varphi} Y_i^F$, where

$$\varphi = \frac{w_i L_i + \Pi_i}{w_i L_i + \Pi_i + R_i} = \frac{\frac{\beta}{1 - \beta} + \frac{\mu}{\sigma} \left(\frac{1}{\gamma^B} + \frac{1 - \beta}{\gamma^S} \right) + \frac{1}{\sigma} \left(1 - \frac{1}{\gamma^B} - \frac{1 - \beta}{\gamma^S} \right)}{\underbrace{\frac{\beta}{1 - \beta} + \frac{\mu}{\sigma} \left(\frac{1}{\gamma^B} + \frac{1 - \beta}{\gamma^S} \right)}_{=w_i L_i / R_i} + \underbrace{\frac{1}{\sigma} \left(1 - \frac{1}{\gamma^B} - \frac{1 - \beta}{\gamma^S} \right)}_{=\Pi_i / R_i} + 1}. \quad (\text{A.47})$$

Then,

$$\begin{aligned} \tilde{R}_i &= \frac{1}{\varphi} Y_i^F + \underbrace{(1 - \beta) (1 + c^S) \sum_d \Lambda_{id} \tilde{R}_d + c^B \tilde{R}_i + (-(1 - \beta) (1 + c^S) - c^B) \tilde{R}_i}_{=0} \\ \iff (1 - c^B) \tilde{R}_i &= (1 - \beta) (1 + c^S) \sum_d \Lambda_{id} \tilde{R}_d + \frac{(1 - (1 - \beta) (1 + c^S) - c^B)}{\varphi} Y_i^F, \end{aligned}$$

where we used the trade balance $\tilde{R}_i = \sum_d \Lambda_{id} \tilde{R}_d$. In matrix notation,

$$\tilde{\mathbf{R}} = \frac{(1 - (1 - \beta)(1 + c^S) - c^B)}{\varphi} [\Phi^{-1}]' \mathbf{Y}^F = \frac{(\beta - (1 - \beta)^{\frac{1-\mu}{\sigma-1}} (\tilde{\lambda}^B + \tilde{\lambda}^S))}{\varphi} [\Phi^{-1}]' \mathbf{Y}^F.$$

Plugging this expression into (A.46),

$$d \ln \mathcal{W} = - \frac{\varphi}{(\beta - (1 - \beta)^{\frac{1-\mu}{\sigma-1}} (\tilde{\lambda}^B + \tilde{\lambda}^S))} \tilde{\mathbf{R}}' (1 - \beta) [\mathbf{\Lambda} \cdot (\boldsymbol{\tau} + \mathbf{Z})]' \mathbf{1}.$$

From the definition of φ in equation (A.47), $\tilde{R}_i \Lambda_{ui} = \frac{1}{1-\varphi} X_{ui}$. Furthermore, $\tilde{R}_i$ is proportional to Y_i^F and $w_i L_i$. Therefore, under the normalization that $\sum_i Y_i^F = 1$, $\tilde{\mathbf{R}}' [\mathbf{\Lambda} \cdot \mathbf{Z}]' = \mathbf{0}$. Hence we have

$$d \ln \mathcal{W} = - \frac{\varsigma}{\left(\frac{\beta}{1-\beta} - \frac{1-\mu}{\sigma-1} (\tilde{\lambda}^B + \tilde{\lambda}^S) \right)} \sum_{u,d} X_{ud} d \ln \tau_{ud}, \quad (\text{A.48})$$

where $\varsigma = \frac{w_i L_i + \Pi_i}{E_i} = \frac{\varphi}{1-\varphi}$ is the ratio of nominal world GDP to nominal world intermediate goods expenditure, as defined in the main text. Hence we obtain the expression in Proposition 3.

Derivation of equation (30) and (31) Reformulating equation (A.43),

$$d \ln C_i = \beta d \ln w_i + (1 - \beta) \sum_u \Lambda_{ui} (d \ln C_u + d \ln \tilde{\tau}_{ui}), \quad (\text{A.49})$$

where

$$d \ln \tilde{\tau}_{ui} \equiv d \ln \tau_{ui} - \frac{1}{\sigma - 1} d \ln M_{ui}. \quad (\text{A.50})$$

In matrix notation,

$$d \ln \mathbf{C} = \tilde{\Phi}^{-1} (1 - \beta) \left[\mathbf{\Lambda} \cdot (\boldsymbol{\tau} + \tilde{\mathbf{Z}}) \right]' \mathbf{1} \quad (\text{A.51})$$

where

$$\tilde{\Phi} = [\mathbf{I} - (1 - \beta) \mathbf{\Lambda}]$$

and $\tilde{\mathbf{Z}}$ are matrices of $\{d \ln \tilde{Z}_{ui}\}$, with

$$d \ln \tilde{Z}_{ui} = \frac{\beta}{1 - \beta} d \ln w_i$$

Together,

$$d \ln \mathcal{W} = - \mathbf{Y}^F' \tilde{\Phi}^{-1} (1 - \beta) \left[\mathbf{\Lambda} \cdot (\tilde{\boldsymbol{\tau}} + \tilde{\mathbf{Z}}) \right]' \mathbf{1}. \quad (\text{A.52})$$

Furthermore,

$$\tilde{R}_i = \frac{1}{\varphi} Y_i^F + (1 - \beta) \underbrace{\sum_d \Lambda_{id} \tilde{R}_d}_{=0} - (1 - \beta) \tilde{R}_i \quad (\text{A.53})$$

$$\iff \tilde{R}_i = (1 - \beta) \sum_d \Lambda_{id} \tilde{R}_d + \frac{\beta}{\varphi} Y_i^F,$$

where we used the trade balance $\tilde{R}_i = \sum_d \Lambda_{id} \tilde{R}_d$. In matrix notation,

$$\tilde{\mathbf{R}} = \frac{\beta}{\varphi} [\tilde{\Phi}^{-1}]' \mathbf{Y}^F. \quad (\text{A.54})$$

Plugging this equation in equation (A.52),

$$d \ln \mathcal{W} = -\frac{\varphi}{\beta} \tilde{\mathbf{R}}' (1 - \beta) [\mathbf{\Lambda} \cdot (\tilde{\tau} + \tilde{\mathbf{Z}})]' \mathbf{1}.$$

From the definition of φ in equation (A.47), $\tilde{R}_i \Lambda_{ui} = \frac{1}{1-\varphi} X_{ui}$. Furthermore, under the normalization that $\sum_i Y_i^F = 1$, $\tilde{\mathbf{R}}' [\mathbf{\Lambda} \cdot \tilde{\mathbf{Z}}]' = \mathbf{0}$. Hence we have

$$d \ln \mathcal{W} = -\frac{\varsigma}{\frac{\beta}{1-\beta}} \sum_{u,d} X_{ud} d \ln \tilde{\tau}_{ud} = -\frac{\varsigma}{\frac{\beta}{1-\beta}} \sum_{u,d} X_{ud} \left(d \ln \tau_{ud} - \frac{1}{\sigma - 1} d \ln M_{ud} \right), \quad (\text{A.55})$$

yielding equation (30).

Equation (31) is obtained immediately by combining equations (27) and (30).

Proposition 3 under optimal sales subsidy Suppose that optimal sales subsidy $s_{id}^I = s^I = 1/\sigma$ are in place as considered in Corollary 1. The only difference from the laissez-faire equilibrium above is the expression for ς , which becomes the ratio of nominal final consumption expenditure (instead of GDP) to the nominal intermediate goods expenditure, such that

$$\begin{aligned} \varsigma^{\text{SalesSubsidy}} &\equiv \frac{w_i L_i + \Pi_i + T_i^F L_i}{E_i} \\ &= \frac{\beta}{1 - \beta} + \frac{1}{1 - s^I} \frac{1}{\sigma} \mu \left(\frac{1}{\gamma^B} + \frac{1 - \beta}{\gamma^S} \right) + \frac{1}{1 - s^I} \frac{1}{\sigma} \left(1 - \frac{1}{\gamma^B} - \frac{1 - \beta}{\gamma^S} \right) - \frac{1}{1 - s^I} \frac{1}{\sigma} \\ &= \frac{\beta}{1 - \beta} - \frac{1 - \mu}{\sigma - 1} \left(\frac{1}{\gamma^B} + \frac{1 - \beta}{\gamma^S} \right) \end{aligned}$$

Reformulating the expression and plugging it into equation (A.55) yields

$$d \ln \mathcal{W}^{\text{SalesSubsidy}} = -\frac{\frac{\beta}{1-\beta} - \frac{1-\mu}{\sigma-1} \left(\frac{1}{\gamma^B} + \frac{1-\beta}{\gamma^S} \right)}{\frac{\beta}{1-\beta} - \frac{1-\mu}{\sigma-1} \left(\tilde{\lambda}^B + \tilde{\lambda}^S \right)} \sum_{u,d} X_{ud} d \ln \tau_{ud}. \quad (\text{A.56})$$

Therefore, when $\lambda^B = 1$ and $\lambda^S = 1 - \beta$, which yield a constrained-efficient equilibrium under the optimal sales subsidy (Corollary 1), the endogenous and fixed networks have identical first-order aggregate effects. When λ^B and λ^S exceed these values, allowing for endogenous networks increases the aggregate effects of shocks; when they are smaller, it decreases them.

It is worth noting that, even if network formation is inefficient, first-order effects of shocks may not be affected by endogenous networks. One such case arises when $\mu = 1$, i.e., labor is the only resource used for relationship formation effort. In this case, although labor may be misallocated between production and relationship-formation activities, there is no margin of distortion for intermediate goods (because they are used solely for production purposes). Another case arises when $\tilde{\lambda}^B + \tilde{\lambda}^S = \frac{1}{\gamma^B} + \frac{1-\beta}{\gamma^S}$ (under optimal sales subsidy). In this case, *relative* efforts are distorted between supplier and buyer link formation, but *total* efforts are not, so aggregate trade flows remain undistorted.

A.8 Proof of Proposition 4

Applying Shephard's Lemma to firm's cost minimization problem (12), and by taking an alternative numeraire such that $w_i = 1$,

$$\begin{aligned} d \ln C_i &= (1 - \beta) \sum_u \Lambda_{ui} (d \ln C_u + d \ln \tau_{ui} - \frac{1}{\sigma - 1} d \ln M_{ui}) \\ &= (1 - \beta) (d \ln C_i - \frac{1}{\sigma - 1} d \ln M_{ii} + \frac{1}{\sigma - 1} d \ln \Lambda_{ii}) \\ &= \frac{1}{\sigma - 1} \frac{1 - \beta}{\beta} d \ln \Lambda_{ii} - \frac{1}{\sigma - 1} \frac{1 - \beta}{\beta} d \ln M_{ii}, \end{aligned} \quad (\text{A.57})$$

where the second transformation used the fact that $\frac{1}{\sigma - 1} d \ln \Lambda_{ii} = -d \ln C_i + \frac{1}{\sigma - 1} d \ln M_{ii} - \sum_u \Lambda_{ui} (-d \ln C_u - d \ln \tau_{ui} + \frac{1}{\sigma - 1} d \ln M_{ui})$.

Notice also that, from cost minimization of retailers (equation 6), $d \ln P_i = d \ln C_i$. Furthermore, firm profit is a constant fraction of labor compensation. Hence $d \ln Q_i^F = -d \ln C_i$.

Furthermore, from Lemmas 1 and 2,

$$\begin{aligned} d \ln M_{ii} &= (\tilde{\lambda}^S + \tilde{\lambda}^B) d \ln X_{ii} - (\tilde{\lambda}^S + \tilde{\lambda}^B) d \ln e_i \\ &= (\tilde{\lambda}^S + \tilde{\lambda}^B) d \ln \Lambda_{ii} - (1 - \mu) (\tilde{\lambda}^S + \tilde{\lambda}^B) d \ln P_i, \end{aligned} \quad (\text{A.58})$$

where the last formulation uses the fact that $d \ln X_{ii} = d \ln \Lambda_{ii} + d \ln E_i = d \ln \Lambda_{ii}$ under our normalization of $w_i = 1$. Combining these together, we have

$$d \ln Q_i^F = - \frac{\frac{1-\beta}{\beta} \frac{1}{\sigma-1} \left(1 - (\tilde{\lambda}^S + \tilde{\lambda}^B) \right)}{1 - \frac{1}{\sigma-1} \frac{1-\beta}{\beta} (1 - \mu) (\tilde{\lambda}^S + \tilde{\lambda}^B)} d \ln \Lambda_{ii}.$$

Manipulating this equation, we obtain the expression in Proposition 4.

B Details on Broader Isomorphisms

This appendix discusses detailed results of Section 5, where we discuss broader isomorphisms and macro restrictions under which our sufficient statistics analysis holds.

B.1 Environment

We first establish a general framework that encompasses different microfoundation of production network formation models. The key generalization of the model in Section 2 is that (1) we allow for arbitrary aggregation of intermediate input bundles (equation B.2 instead of 2) and (2) we do not impose a structure of the network formation $\{\mathcal{S}_{ui}(\cdot)\}$ as specified in Section 2.2.

Firms. A continuum of firms produces a distinct variety in each location. Production of intermediate goods requires labor and intermediate inputs. Intermediate inputs are sourced from firms that are directly connected by production networks. The production function of firm $\omega \in \Omega_i$ is given by

$$q_i(\omega) = z_i(\omega) \left(\frac{l_i(\omega)}{\beta} \right)^\beta \left(\frac{\tilde{q}_i(\omega)}{1-\beta} \right)^{1-\beta}, \quad (\text{B.1})$$

where $z_i(\omega)$ is the total factor productivity (TFP) of firm ω which follows distribution $G_i(\cdot)$, $l_i(\omega)$ is labor inputs, and $\tilde{q}_i(\omega)$ is the composite of intermediate inputs, β is the parameter proxying the input share for labor, given by

$$\tilde{q}_i(\omega) = F_i \left(\{q_{ui}(v, \omega)\}_{v \in \mathcal{S}_{ui}(\omega), u \in \mathcal{N}} \right). \quad (\text{B.2})$$

Given these production functions, the marginal cost of production by firm ω is given by

$$c_i(\omega) = \frac{1}{z_i(\omega)} w_i^\beta \tilde{F}_i \left(\{p_{ui}(v, \omega)\}_{v \in \mathcal{S}_{ui}(\omega), u} \right), \quad (\text{B.3})$$

where $p_{ui}(v, \omega)$ is the intermediate goods price that supplier v in location u charges to firm ω in location i . On top of these production costs, when a firm sells its intermediate goods to location d , it incurs an iceberg trade cost of $\tau_{id} \geq 1$.

We assume that suppliers charge a constant markup of their marginal cost net of the iceberg trade cost

$$p_{id}(v, \omega) = \varrho_{id}(v, \omega) c_i(v) \tau_{id}, \quad (\text{B.4})$$

where $\varrho_{id}(v, \omega)$ is the markup ratio.

Retailers. Perfectly competitive retailers in each location i combine intermediate inputs from all firms in location i and produce a standardized nontradable retail good. Their production function is given by

$$Q_i = g_i \left(\{q_i^R(\omega)\}_{\omega \in \Omega_i} \right), \quad (\text{B.5})$$

where $g_i(\cdot)$ is a function that satisfies homogeneous of degree one, and $q_i^R(\omega)$ is the quantity of intermediate inputs from firm ω .

We also assume that retailers have the entire bargaining power when purchasing intermediate inputs from each firm at their marginal cost $c_i(v)$. Under cost minimization, retail prices are given by

$$P_i = \tilde{g}_i(\{c_i(\omega)\}_{\omega \in \Omega_i}), \quad (\text{B.6})$$

where $\tilde{g}_i(\cdot)$ is a solution to the cost minimization problem by retailers.

Final Consumers. Measure L_i Households supply labor inelastically at wage w_i . They also own an equal share of local firms. Therefore, their budget constraint is given by

$$P_i Q_i^F = w_i + \frac{\Pi_i}{L_i}, \quad (\text{B.7})$$

where Q_i^F is the amount of consumption of retail goods per capita, and Π_i is aggregate profit by firms producing in location i .

Production Network Formation. Next we describe how the production network structure, $\{\mathcal{S}_{ui}(\cdot)\}$, is endogenously determined. Existing papers have taken different approaches to these networks, such as through the entry decisions into a market or forming a relationship, search and matching decisions, or a choice of suppliers. Here, we simply assume that they are produced by combining labor and retail goods by firms in upstream locations and downstream locations. We denote that there are aggregate relationships between the profiles of labor and retail goods by firms in the upstream and downstream locations, such that

$$\{\mathcal{S}_{ui}(\cdot)\} = G_{ui}\left(\{l_u^U(\omega)\}_{\omega \in \Omega_u}, \{Q_u^U(\omega)\}_{\omega \in \Omega_u}, \{l_d^D(\omega)\}_{\omega \in \Omega_d}, \{Q_d^D(\omega)\}_{\omega \in \Omega_d}\right) \quad (\text{B.8})$$

Market clearing. We introduce various market-clearing conditions to close the model. Intermediate goods for each firm ω are used as inputs for production by other firms and local retailers:

$$q_i^R(\omega) + \sum_d \int_{\psi: \omega \in S_{id}(\psi)} q_{id}(\omega, \psi) \tau_{id} = q_i(\omega). \quad (\text{B.9})$$

Retail goods are consumed by final consumers and used for relationship-formation activity:

$$L_i Q_i^F + \int_{\omega \in \Omega_i} Q_i^U(\omega) d\omega + \int_{\omega \in \Omega_i} Q_i^D(\omega) d\omega = Q_i. \quad (\text{B.10})$$

Labor is used for intermediate goods production and for relationship-formation activity:

$$\int_{\omega \in \Omega_i(\omega)} l_i(\omega) d\omega + \int_{\omega \in \Omega_i} l_i^U(\omega) d\omega + \int_{\omega \in \Omega_i} l_i^D(\omega) d\omega = L_i. \quad (\text{B.11})$$

The general equilibrium is defined as follows:

Definition B.1. The equilibrium is defined by the set of prices $\{p_{id}(v, \omega), P_i, e_i, w_i, C_i\}$ and quantities $\{q_{id}(\omega, \psi), q_i^R(\omega), Q_i^F, Q_i^A, Q_u^U(\omega), Q_d^D(\omega), l_i(\omega), l_u^U(\omega), l_d^D(\omega), L_i^A, \tilde{\pi}_i(\omega)\}$ that satisfy the following set of conditions:

1. Households maximize consumption given the budget constraint (7) with income from firm profit given by Π_i
2. Firms make optimal pricing and production decisions for intermediate goods (3), (4); retailers make optimal production decisions for retail goods (6)
3. The structure of the network follows (B.8), with some rules to determine $\{l_u^U(\omega)\}_{\omega \in \Omega_u}$, $\{Q_u^U(\omega)\}_{\omega \in \Omega_u}$, $\{l_d^D(\omega)\}_{\omega \in \Omega_d}$, $\{Q_d^D(\omega)\}_{\omega \in \Omega_d}$.
4. All markets clear (B.9), (B.10), and (B.11)

B.2 General Results

We first introduce three macro restrictions extending Arkolakis et al. (2012) to an environment with endogenous production networks.

Condition B.1. *Aggregate trade balance holds, $\sum_i X_{ui} = \sum_d X_{id}$ for all i .*

Condition B.2. *Aggregate profit Π_i and labor compensation $w_i L_i$ are constant fraction of aggregate intermediate goods sales (excluding those to retailers) R_i for all i . We denote GDP-to-intermediate-goods-expenditure ratio $\varsigma \equiv (w_i L_i + \Pi_i) / R_i$.*

Condition B.3. *Aggregate trade elasticity is CES. That is, $\frac{\partial \ln X_{ui}/X_{ii}}{\partial \ln \tau_{ui}} = \varepsilon$ for all u, i .*

The three conditions are the same as Arkolakis et al. (2012) except that Condition B.2 is extended to an environment with intermediates. In addition, we introduce two more macro restrictions that are relevant for the endogenous production networks.

Condition B.4. *Denote the aggregate measure of links by $M_{ui} = \int_{\omega \in \Omega_i} S_{ui}(\omega) d\omega$. Then, it follows the following equilibrium relationships:*

$$d \ln M_{ui} = (\delta_{L,U} + \delta_{Q,U} + \delta_{L,D} + \delta_{Q,D}) d \ln X_{ui} - \delta_{L,U} d \ln w_u - \delta_{Q,U} d \ln P_u - \delta_{L,D} d \ln w_d - \delta_{Q,D} d \ln P_d \quad (\text{B.12})$$

Condition B.5. *The retail goods cost has the following relationships*

$$d \ln P_i = \beta d \ln w_i + (1 - \beta) \sum_u \Lambda_{ui} (d \ln P_u + d \ln \tau_{ui} - \nu d \ln M_{ui}) \quad (\text{B.13})$$

where ν is some constant parameter.

The following proposition shows that versions of Propositions 3 and 4 hold if these macro restrictions are satisfied.

Proposition B.1. Consider global welfare with GDP weights: $\psi_i = w_i L_i + \Pi_i$. Under Conditions B.1- B.5, the first-order effect of a shock in iceberg trade costs $\{d \ln \tau_{ij}\}$ on global welfare is given by:

$$d \ln \mathcal{W} = \frac{\varsigma}{\frac{\beta}{1-\beta} - \nu (\delta_{Q,U} + \delta_{Q,D})} \sum_{u,d} X_{ud} d \ln \tau_{ud}, \quad (\text{B.14})$$

where ς is the ratio of nominal world GDP to nominal world intermediate goods expenditure by firms defined in Condition B.2.

Proposition B.2. If Conditions B.1- B.5 are satisfied, the welfare changes in location i from external shocks of any magnitude is given by:

$$d \ln Q_i^F = -\frac{1}{\varepsilon \frac{\beta}{1-\beta} - \nu (\delta_{Q,U} + \delta_{Q,D})} d \ln \Lambda_{ii}, \quad (\text{B.15})$$

The proof of this proposition, as well as the one below, follows a similar structure as the proof of Proposition 3 (by replacing equations A.44 and A.43 with equations B.12 and B.13) and Proposition 4 (by replacing equations A.58 and A.57 with equations B.12 and B.13), and hence omitted here.

B.3 Specific Cases

B.3.1 Search and Matching

We start by discussing that our baseline model satisfies Conditions B.1-B.5. Condition B.1 is trivial from the market clearing conditions (see Footnote 1). Condition B.2 holds because our model predicts constant aggregate profit and labor share, as discussed in Section 2.2. Finally, Condition B.3 holds as with ε given by equation (19). Condition B.4 is satisfied by the isoelastic relationship-formation decisions (8) and Cobb-Douglas matching technology (16), with $\delta_{L,U} = \mu \tilde{\lambda}^B$, $\delta_{Q,U} = (1 - \mu) \tilde{\lambda}^B$, $\delta_{L,D} = \mu \tilde{\lambda}^S$, and $\delta_{Q,D} = (1 - \mu) \tilde{\lambda}^S$. Condition B.5 holds from cost minimization (12) with $\nu = 1/(\sigma - 1)$, which captures the degree of love of variety in intermediate inputs.

We also argue that the macro restrictions hold in the alternative specifications where either supplier or buyer relationship-formation effort is undirected with respect to locations. First, consider the case where buyer relationship-formation effort is undirected. The relationship-formation costs are modified from equation (8) to

$$f_i(n_i^B, \{n_{ui}^S\}_u) = e_i \left\{ f_i^B \frac{(n_i^B)^{\gamma^B}}{\gamma^B} + \sum_{u \in \mathcal{N}} f_{ui}^S \frac{(n_{ui}^S)^{\gamma^S}}{\gamma^S} \right\},$$

where notice that buyer relationship-formation effort n_i^B does not depend on destinations d . The matching technology is modified from equation (16) to

$$M_{ud} = \kappa_{ud} \left(\overline{M}_u^B \right)^{\lambda^B} \left(\overline{M}_{ud}^S \right)^{\lambda^S},$$

where $\overline{M}_u^B = N_u \int_z n_u^B(z) dG_u(z)$ and $\overline{M}_{ud}^S = N_d \int_z n_{ud}^S(z) dG_d(z)$. Given M_{ud} , the matching rates m_u^S and m_u^B are determined as:

$$m_u^B = \sum_d \frac{M_{ud}}{\overline{M}_u^B}, \quad m_{ud}^S = \frac{M_{ud}}{\overline{M}_{ud}^S}.$$

Following the same algebra as in Lemmas 1 and 2, we can verify that this model satisfies the macro restrictions with $\varepsilon = \frac{\sigma-1}{1-\tilde{\lambda}^S}$, $\delta_{L,U} = \mu \tilde{\lambda}^B$, $\delta_{Q,U} = (1-\mu) \tilde{\lambda}^B$, $\delta_{L,D} = \mu \tilde{\lambda}^S$, $\delta_{Q,D} = (1-\mu) \tilde{\lambda}^S$, and $\nu = 1/(\sigma-1)$. Therefore, the only difference in our welfare sufficient statistics (38) and (39) arises through the differences in trade elasticity ε . Importantly, trade elasticity is smaller than our baseline model, as it only depends on the matching elasticities on the supplier side.

The case where the supplier relationship-formation effort is undirected can be considered analogously. In this case, this model satisfies the macro restrictions with $\varepsilon = \frac{\sigma-1}{1-\tilde{\lambda}^B}$, $\delta_{L,U} = \mu \tilde{\lambda}^B$, $\delta_{Q,U} = (1-\mu) \tilde{\lambda}^B$, $\delta_{L,D} = \mu \tilde{\lambda}^S$, $\delta_{Q,D} = (1-\mu) \tilde{\lambda}^S$, and $\nu = 1/(\sigma-1)$.

Finally, we discuss the case where the relationship-formation effort is directed toward a specific firm productivity segment. Specifically, we assume that firms with productivity φ_i in location i decide a relationship-formation effort $\{n_{id}^B(\varphi_i, \varphi_d)\}_{d, \varphi_d}$ for buyers with productivity φ_d in location d . Similarly, they make a relationship-formation effort $\{n_{ui}^S(\varphi_u, \varphi_i)\}_{u, \varphi_u}$ for suppliers with productivity φ_u in location u . We assume the total relationship-formation cost takes the following form:

$$e_i \left(\sum_d \int f_{id}^B(\varphi_i, \varphi_d) n_{id}^B(\varphi_i, \varphi_d)^{\gamma^B} dG_d(\varphi_d) + \sum_u \int f_{ui}^S(\varphi_u, \varphi_i) n_{ui}^S(\varphi_u, \varphi_i)^{\gamma^S} dG_u(\varphi_u) \right)$$

where $f_{id}^B(\varphi_i, \varphi_d)$ and $f_{ui}^S(\varphi_u, \varphi_i) n_{ui}^S(\varphi_u, \varphi_i)$ denote the relationship-formation cost shifters that depend on the suppliers' and buyers' locations and productivity. We assume that the matching market clears for each pair of location and productivity, such that the total links between them is given by

$$M_{ud}(\varphi_u, \varphi_d) = \kappa_{ud}(\varphi_u, \varphi_d) \left(\overline{M}_u^B(\varphi_u, \varphi_d) \right)^{\lambda^B} \left(\overline{M}_{ud}^S(\varphi_u, \varphi_d) \right)^{\lambda^S},$$

where $\overline{M}_u^B(\varphi_u, \varphi_d)$ and $\overline{M}_{ud}^S(\varphi_u, \varphi_d)$ are aggregate efforts for buyer and supplier link formation, respectively.

This case delivers identical expressions for the aggregate welfare with our baseline model with $\delta_{L,U} = \mu \tilde{\lambda}^B$, $\delta_{Q,U} = (1-\mu) \tilde{\lambda}^B$, $\delta_{L,D} = \mu \tilde{\lambda}^S$, and $\delta_{Q,D} = (1-\mu) \tilde{\lambda}^S$. This isomorphism arises because (location-pair-specific) trade cost shocks do not generate any reallocation of efforts across firm segments.

B.3.2 Relationship-Specific Fixed Cost

This section develops a production network formation model with relationship-specific fixed cost incurred by suppliers. The structure of production given networks follow exactly as in Section 2.1. In addition, we assume that the productivity $z_i(\omega)$ follows Pareto distribution with shape parameter θ with lower bound z^* . In terms of network formation, we assume that any pair of a

supplier in u and a buyer in d form a relationship as long as the supplier is willing to pay a fixed cost f_{ud} . These fixed cost are paid as a Cobb-Douglas composite of supplier's location's labor and retail goods with labor share μ , similarly to the relationship-formation costs in our baseline specification.

Let us conjecture that firm's production cost and intermediate goods revenue take the following form:

$$c_i(z) = C_i z^{-\gamma_C}, \quad r_i(z) = R_i z^{\gamma_R},$$

where γ_C, γ_R are constants, and C_i and R_i are location-level cost and revenue shifter. Furthermore, conjecture that the upper-bound of the marginal cost of firms in location u to supply to a firm in location i with productivity z (by paying a fixed cost) takes the following form:

$$\bar{c}_{ui}(z) = \Omega_{ui} z^{\gamma_P},$$

where γ_P is a constant. We now confirm these conjectures are correct and derive the expressions for $\gamma_C, \gamma_R, \gamma_P$.

First, from cost minimization,

$$c_i(z) = \frac{1}{z} w_i^\beta \tilde{p}_i(z)^{1-\beta}, \quad \tilde{p}_i(z) = \left(\sum_u \left(\frac{\sigma}{\sigma-1} \bar{c}_{ui}(z) \right)^{\gamma_C \theta} \int_{p \leq \frac{\sigma}{\sigma-1} \bar{c}_{ui}(z)} p^{1-\sigma} dG_c(p) \right)^{\frac{1}{1-\sigma}} \quad (\text{B.16})$$

where $G_c(\cdot)$ is an inverse Pareto distribution with lower bound $\frac{\sigma}{\sigma-1} \bar{c}_{ui}(z)$ and shape parameter $\gamma_C \theta$.

Next, we consider the zero-profit condition for a marginal supplier to sell to a buyer. Denote the revenue by a supplier with marginal cost c to a buyer with productivity z , conditional on a positive transaction, by

$$r_{ui}(c \tau_{ui}, z) = \frac{\left(\frac{\sigma}{\sigma-1} c \tau_{ui} \right)^{1-\sigma}}{\tilde{p}_i(z)^{1-\sigma}} r_i(z)$$

Similarly to [Bernard et al. \(2018\)](#), we consider a limit where the lower-bound of productivity $z^* \rightarrow 0$. Then, from the zero-profit condition of a marginal supplier,

$$r_{ui}(\bar{c}_{ui}(z) \tau_{ui}, z) = \frac{\left(\frac{\sigma}{\sigma-1} \bar{c}_{ui}(z) \tau_{ui} \right)^{1-\sigma}}{\tilde{p}_i(z)^{1-\sigma}} r_i(z) = f_{ui} w_u^\mu P_u^{1-\mu} \quad (\text{B.17})$$

Finally, from the definition of intermediate firm revenue,

$$r_u(z) = r_u^F(z) + \sum_d N_d \int_{c_u(z) \leq \bar{c}_{ui}(z')} r_{ud}(c_u(z) \tau_{ud}, z') dG_d(z') \quad (\text{B.18})$$

where $r_u^F(z)$ is sales to local retailers, and N_d proxies the measure of firms in location d . To obtain closed form solution, we assume that retail goods production technology (B.5) takes the form of CES with the elasticity of substitution γ_R , such that $r_u^F(z) = R_u^F z^{\gamma_R}$.

By focusing on the exponents on z , it can be shown that equations (B.16), (B.17), (B.18) are

satisfied with

$$\begin{aligned}\gamma_C &= \frac{1 - (1 - \beta)}{1 - (1 - \beta) \frac{\theta}{\sigma - 1}} > 1 \\ \gamma_P &= 1 \\ \gamma_R &= \gamma_C \theta\end{aligned}$$

Notice now that the aggregate expenditure share is given by

$$\Lambda_{ui} = \frac{\int_{c_u(z) \geq \bar{c}_{ui}(z')} r_{ud}(c_u(z) \tau_{ud}, z') dG_d(z')}{\sum_u \int_{c_u(z) \geq \bar{c}_{ui}(z')} r_{ud}(c_u(z) \tau_{ud}, z') dG_d(z')} = \frac{\bar{c}_{ui}(z')^{\gamma_C \theta}}{\sum_u \bar{c}_{ui}(z')^{\gamma_C \theta}}$$

Hence Condition (B.3) holds with

$$\varepsilon = \gamma_C \theta.$$

Furthermore, from equation (B.16),

$$\nu = \frac{1}{\sigma - 1} - \frac{1}{\gamma_C \theta}.$$

It is also straightforward to verify that the other conditions hold.

B.3.3 Discrete Choice of Suppliers

In this section, we discuss a version of [Oberfield \(2018\)](#) and [Eaton et al. \(2024\)](#). Firms choose the optimal supplier among the randomly drawn potential suppliers. Suppliers provide homogeneous products for each task. Therefore, production technology (B.2) can be given by

$$\tilde{q}_i(\omega) = \sum_u \sum_{v \in S_{ui}(\omega)} q_{ui}(v, \omega).$$

Hence the marginal cost of production (B.3) is given by

$$c_i(\omega) = \frac{1}{z_i(\omega)} w_i^\beta \left(\min_{v \in S_{ui}(\omega)} c_u(v) \tau_{ui} \right)^{1-\beta}.$$

We also assume that the set of suppliers $S_{ui}(\omega)$ is determined simply by exogenous random process, i.e., probability of supplier in u to match with i is given by a Poisson process with exogenous location-pair-specific parameter.²

As [Oberfield \(2018\)](#) and [Eaton et al. \(2024\)](#), this model yields a tractable solution if we assume the Pareto productivity distribution. In particular, aggregate expenditure share is given by

$$\Lambda_{ui} = \frac{(P_u \tau_{ui})^\theta}{\sum_u (P_u \tau_{ui})^\theta}$$

²[Eaton et al. \(2024\)](#) considers a set up where the matching rate depends on the supplier's production cost. In this case, the aggregate trade elasticity is affected by how the matching rate varies by the supplier's production efficiency. Furthermore, we abstract in-house production for intermediate goods, which leads to a violation of the constant labor share restriction.

Therefore, Condition B.3 holds with

$$\varepsilon = \theta$$

Furthermore, given that $S_{ui}(\omega)$ is determined simply by an exogenous random process without using resource costs, Condition B.3 holds with

$$\delta_{L,U} = \delta_{Q,U} = \delta_{L,D} = \delta_{Q,D} = 0$$

It is also straightforward to verify that the other conditions hold.

C Multiple Sectors

This appendix develops a model of endogenous production network formation across space and sectors.

The economy is segmented by a finite number of locations denoted by $u, i, d \in \mathcal{N}$. In each location, there is an L_i measure of households. Each household supplies one unit of labor inelastically and earns a competitive wage w_i . There is a fixed mass of intermediate goods producers in each location, which we call “firms” in short. Firms also belong to a sector, denoted by $h, k, l \in K$. We denote each firm in location i and sector k by $\omega \in \Omega_{i,k}$ and the measure of firms in location i and sector k by $N_{i,k}$. We denote the distribution of total factor productivity (TFP) by firms in location i and sector k by $G_{i,k}(\cdot)$, which can flexibly depend on the location. Firms produce differentiated intermediate goods combining labor and intermediate goods. Intermediate goods can be traded across firms in different locations, subject to iceberg trade costs, as long as there are production linkages between them. Local competitive retailers sell the combined composites to local consumers.

We model production networks as the presence of connections between supplier firms and buyer firms. Specifically, we denote $S_{ui,hk}(\omega) \subseteq \Omega_{u,h}$ to indicate the set of suppliers producing in u that a firm ω in location i can purchase from. In what follows, we first describe how production occurs given networks $\{S_{ui,hk}(\cdot)\}_{u,i,h,k}$. We then describe how these networks are endogenously formed through a search and matching process.

C.1 Production given Networks

Firms. A continuum of firms produces a distinct variety in each location and sector. Production of intermediate goods requires labor and intermediate inputs. Intermediate inputs are sourced from firms that are directly connected by production networks. The production function of firm $\omega \in \Omega_{i,k}$ is given by

$$q_{i,k}(\omega) = z_{i,k}(\omega) \left(\frac{l_{i,k}(\omega)}{\beta_{k,L}} \right)^{\beta_{k,L}} \prod_h \left(\frac{\tilde{q}_{i,hk}(\omega)}{\beta_{hk}} \right)^{\beta_{hk}}, \quad (\text{C.1})$$

where $z_{i,k}(\omega)$ is the total factor productivity (TFP) of firm ω which follows distribution $G_{i,k}(\cdot)$, $l_{i,k}(\omega)$ is labor inputs, and $\tilde{q}_{i,hk}(\omega)$ is the composite of intermediate inputs, $\beta_{k,L}$ is the parameter proxying the input share for labor, and β_{hk} is the parameter proxying the input share for intermediate inputs from sector h . We assume that the production technology is constant returns to scale such that $\beta_{s,L} + \sum_{k \in K} \beta_{ks} = 1$. The composite of intermediate inputs is a constant elasticity of substitution (CES) aggregator of the input varieties sourced from their connected suppliers. Therefore,

$$\tilde{q}_{i,hk}(\omega) = \left(\sum_u \int_{v \in S_{ui,hk}(\omega)} q_{ui,hk}(v, \omega)^{\frac{\sigma_h - 1}{\sigma_h}} dv \right)^{\frac{\sigma_h}{\sigma_h - 1}}, \quad (\text{C.2})$$

where $q_{ui,hk}(v, \omega)$ is the quantity of input for each variety, and σ_h is the elasticity of substitution.

Given these production functions, the marginal cost of production by firm ω is given by

$$c_{i,k}(\omega) = \frac{1}{z_{i,k}(\omega)} w_i^{\beta_{k,L}} \prod_h \left(\sum_u \int_{v \in S_{ui,hk}(\omega)} p_{ui,hk}(v, \omega)^{1-\sigma_h} dv \right)^{\frac{\beta_{hk}}{1-\sigma_h}}, \quad (\text{C.3})$$

where $p_{ui,hk}(v, \omega)$ is the intermediate goods price that supplier v in location u and sector h charges to firm ω in location i and sector k . On top of these production costs, when a firm sells its intermediate goods to location d and sector l , it incurs an iceberg trade cost of $\tau_{id,kl} \geq 1$.

We assume that all firms are matched with a continuum of suppliers. Furthermore, suppliers are under monopolistic competition to supply to each buyer. Thus, given the isoelastic intermediate goods demand (equation C.3), suppliers charge a constant markup of their marginal cost net of the iceberg trade cost;

$$p_{id,kl}(v, \omega) = \tilde{\sigma}_k c_{i,k}(v) \tau_{id,kl}, \quad (\text{C.4})$$

where $\tilde{\sigma}_k = \sigma_k / (\sigma_k - 1)$ is the markup ratio.

Notice that firms producing in the same location with the same productivity $z_{i,k}(\omega)$ charge the same prices and earn the same profit. As we describe below, they also make the same decisions regarding supplier and buyer relationship-formation effort. Therefore, without risk of confusion, we sometimes index the cost function using z instead of ω , e.g., $c_{i,k}(z)$ instead of $c_{i,k}(\omega)$ for firm ω whose productivity is $z = z_{i,k}(\omega)$.

Retailers. Perfectly competitive retailers in each location i and sector k combine intermediate inputs from all firms in location i and sector k and produce a standardized nontradable retail good. Their production function is given by

$$Q_{i,k} = g_{i,k} \left(\{q_{i,k}^R(\omega)\}_{\omega \in \Omega_{i,k}} \right), \quad (\text{C.5})$$

where $g_{i,k}(\cdot)$ is a function that satisfies homogeneous of degree one, and $q_{i,k}^R(\omega)$ is the quantity of intermediate inputs from firm ω . The retail goods are used for final consumption purposes and by relationship-formation activity, as we describe further below.

We also assume that retailers have the entire bargaining power when purchasing intermediate inputs from each firm at their marginal cost $c_{i,k}(v)$. Under cost minimization, retail prices are given by

$$P_{i,k} = \tilde{g}_{i,k} \left(\{c_{i,k}(\omega)\}_{\omega \in \Omega_{i,k}} \right), \quad (\text{C.6})$$

where $\tilde{g}_{i,k}(\cdot)$ is a solution to the cost minimization problem by retailers.

Final Consumers. Measure L_i Households supply labor inelastically at wage w_i . They also own an equal share of local firms. Therefore, their budget constraint is given by

$$P_i Q_i^F = w_i + \frac{\sum_k \Pi_{i,k}}{L_i}, \quad (\text{C.7})$$

where $\Pi_{i,k}$ is aggregate profit by firms producing in location i . Q_i^F is a Cobb-Douglas aggregator of retail goods across sector, i.e.,

$$Q_i^F = \prod_k (Q_{i,k}^F)^{\alpha_k}. \quad (\text{C.8})$$

C.2 Production Network Formation

We now describe how the production network structure, $\{\mathcal{S}_{ui,hk}(\cdot)\}$, emerges endogenously from the network formation decisions of profit-maximizing firms subject to matching frictions.

C.2.1 Firms' Relationship-Formation Decisions

We assume that firms pay upfront effort in forming supplier and buyer relationships. These relationship-formation costs capture the various costs involved in establishing successful trade links, such as identifying counterparts and designing or customizing products. The probability that a relationship materializes successfully depends jointly on the resources spent for relationship formation by both suppliers and buyers, given the matching technology.

Each firm in location i chooses the effort to form supplier relationships across origins, $\{n_{ui,hk}^S\}_{u,h}$, and in forming buyer relationships across destinations, $\{n_{id,kl}^B\}_{d,l}$. We assume that these relationship formation decisions entail isoelastic, upward-sloping costs. The total cost for relationship formation is

$$f_{i,k}(\{n_{id,kl}^B\}_{d,l}, \{n_{ui,hk}^S\}_{u,h}) = e_{i,k} \left\{ \sum_{d \in \mathcal{N}} \sum_l f_{id,kl}^B \frac{(n_{id,kl}^B)^{\gamma_k^B}}{\gamma_k^B} + \sum_{u \in \mathcal{N}} \sum_h f_{ui,hk}^S \frac{(n_{ui,hk}^S)^{\gamma_k^S}}{\gamma_k^S} \right\}, \quad (\text{C.9})$$

where $e_{i,k}$ is the unit cost of resources put for relationship formation in location i and sector k (we describe how this is determined at the end of this section), and γ_k^B and γ_k^S are parameters capturing the decreasing returns in relationship-formation efforts. $\{f_{id,kl}^B\}$ and $\{f_{ui,hk}^S\}$ are location-sector-pair-specific relationship-formation cost shifters, capturing the possibility that the cost of identifying and forming supplier and buyer relationships may depend on geographic frictions.

A unit effort for supplier link formation, $\{n_{ui,hk}^S\}_u$, turns into successful supplier relationships at location-sector-pair specific match rate $\{m_{ui,hk}^S\}_{u,h}$. Similarly, a unit effort for buyer link formation, $\{n_{id,kl}^B\}_{d,l}$, turn into successful buyer relationships at rate $\{m_{id,kl}^B\}_{d,l}$. In the equilibrium, these matching rates are endogenously determined by the aggregate bilateral efforts made by suppliers and buyers and matching technology, as we describe in the next section. However, each firm is atomistic and hence takes these matching rates as given.

We now define firms' relationship-formation decisions. Given the random matching with suppliers, and the cost function (3), the intermediate goods cost is affected by supplier relationship-formation decisions $\{n_{ui}^S\}_u$ as follows:

$$c_{i,k}(z, \{n_{ui,hk}^S\}_{u,h}) = \frac{1}{z} w_{i,k}^{\beta_{k,L}} \prod_h \left(\sum_{u \in \mathcal{N}} n_{ui,hk}^S m_{ui,hk}^S C_{ui,hk}^{1-\sigma_h} \right)^{\frac{\beta_{hk}}{1-\sigma_h}}, \quad (\text{C.10})$$

where $C_{ui,hk}^{1-\sigma_h} \equiv \int (\tilde{\sigma}_h c_{u,h}(z) \tau_{ui,hk})^{1-\sigma_h} dG_{ui,hk}^B(z)$ is the CES aggregator of the price of a supplier producing in location u in sector h to supply to location i and sector k , and $G_{ui,hk}^B(z)$ is the distribution of productivity weighted by the buyer relationship-formation effort. Depending on this cost, the firm in location i with productivity z 's expected profit per buyer in location d is given by $\frac{1}{\sigma_k} (\tilde{\sigma}_k \tau_{id,kl} c_i(z, \{n_{ui,hk}^S\}_{u,h}))^{1-\sigma_k} D_{d,kl}$, where $D_{d,kl}$ is the average demand net of the price index averaged across all buyers. Together, the optimal relationship-formation decisions for $\{n_{ui,hk}^S\}_u, \{n_{id,kl}^B\}_{d,l}$ are given by:

$$\begin{aligned} \tilde{\pi}_{i,k}(\omega) \equiv & \max_{\{n_{id,kl}^B\}_{d,l}, \{n_{ui,hk}^S\}_{u,h}} \sum_{l,d} m_{id,kl}^B n_{id,kl}^B \frac{1}{\sigma_k} (\tilde{\sigma}_k \tau_{id,kl} c_{i,k}(z, \{n_{ui,hk}^S\}_{u,h}))^{1-\sigma_k} D_{d,kl} \\ & - e_{i,k} \left\{ \sum_{d \in \mathcal{N}} \sum_l f_{id,kl}^B \frac{(n_{id,kl}^B)^{\gamma_k^B}}{\gamma_k^B} + \sum_{u \in \mathcal{N}} \sum_h f_{ui,hk}^S \frac{(n_{ui,hk}^S)^{\gamma_k^S}}{\gamma_k^S} \right\}, \end{aligned} \quad (\text{C.11})$$

subject to (C.10).

The first term inside the max operator represents the firm profit, and the last term is the relationship-formation cost, as discussed above. We impose a parameter restriction that $1 - \frac{1}{\gamma_k^B} - \frac{\sum_h \beta_{hk}}{\gamma_k^S} > 0$, which guarantees that firms make positive sales and profit. The following lemma characterizes the solution to this problem.

Lemma C.1. *The solution to the supplier and buyer relationship-formation problem (C.11) is given by*

$$n_{id,kl}^B(z) = a_{id,kl}^B z^{\frac{\delta_k}{\gamma_k^B}}; \quad n_{ui,hk}^S(z) = a_{ui,hk}^S z^{\frac{\delta_k}{\gamma_k^S}}, \quad (\text{C.12})$$

where $\delta_k \equiv (\sigma_k - 1) / \left(1 - \frac{1}{\gamma_k^B} - \frac{1}{\gamma_k^S} \sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}\right)$, where

$$a_{id,kl}^B = \left(\Gamma_{i,k}^B \frac{X_{id,kl}}{e_{i,k} f_{id,kl}^B} \right)^{\frac{1}{\gamma_{hk}^B}}, \quad a_{ui,hk}^S = \left(\Gamma_{i,hk}^S \frac{R_{i,k}}{E_{i,k}} \frac{X_{ui,hk}}{e_{i,k} f_{ui,hk}^S} \right)^{\frac{1}{\gamma_k^S}} \quad (\text{C.13})$$

where $\Gamma_{i,k}^B = \frac{1}{\sigma_k \mathbb{M}_{i,k}(\delta_k) N_{i,k}}$ and $\Gamma_{i,hk}^S = \frac{1}{\sigma_k \mathbb{M}_{i,k}(\delta_k) N_{i,k}} \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}$, and $X_{id,kl}$ is total nominal trade flows from i, k to d, l .

Furthermore, the unit cost of firms, $c_i(z)$, can be expressed as

$$c_{i,k}(z) = C_{i,k} z^{-\frac{\delta_k}{\gamma_k^S} \sum_h \frac{\beta_{hk}}{\sigma_h - 1} - 1}, \quad (C_{i,k})^{1-\sigma_k} \equiv w_i^{\beta_{k,L}(1-\sigma_k)} \prod_h \left(\sum_{u \in \mathcal{N}} a_{ui,hk}^S m_{ui,hk}^S (C_{ui,hk})^{1-\sigma_h} \right)^{\frac{\beta_{hk}}{1-\sigma_h} (1-\sigma_k)}, \quad (\text{C.14})$$

Total revenue from sales to other firms (excluding sales to retailers) is

$$r_{i,k}(z) = (\tilde{\sigma}_k)^{1-\sigma_k} D_{i,k}^* (C_{i,k})^{1-\sigma_k} (z)^{\delta_k}, \quad D_{i,k}^* = \sum_{d \in \mathcal{N}} \sum_l m_{id,kl}^B a_{id,kl}^B D_{d,kl} (\tau_{id,kl})^{1-\sigma_k}. \quad (\text{C.15})$$

Search intermediaries. Finally, we describe how the unit cost of relationship-formation effort, $e_{i,k}$, is determined. In practice, forming production networks entails diverse types of activities, such as advertising and sales promotion, identifying potential partners, developing prototypes, and designing or customizing products, which require both labor and material inputs. To capture these dimensions, we model the relationship-formation costs as a Cobb–Douglas aggregator of labor and retail goods. Under cost minimization, the unit cost of relationship formation is given by:

$$e_{i,k} = (w_i)^{\mu_k} (P_{i,k})^{1-\mu_k}. \quad (\text{C.16})$$

where μ_k is the labor share in relationship-formation cost.

C.2.2 Matching Technology

We follow the labor search-and-matching literature and represent the process through an aggregate matching technology that links the bilateral efforts of relationship formation by the suppliers and buyers to the measure of successful relationships (Diamond 1982; Mortensen 1986; Pissarides 1985). Specifically, we assume that the aggregate number of successful matches between locations, $M_{ud,hl}$, is governed by a Cobb–Douglas matching function:

$$M_{ud,hl} = \kappa_{ud,hl} \left(\overline{M}_{ud,hl}^S \right)^{\lambda_{hl}^S} \left(\overline{M}_{ud,hl}^B \right)^{\lambda_{hl}^B}, \quad (\text{C.17})$$

where $\lambda_{hl}^B, \lambda_{hl}^S \geq 0$ denote the elasticities of total matches created for the pair of regions with respect to the aggregate effort for buyer and supplier link formation, respectively;³ $\kappa_{ud,hl}$ is the parameter governing the efficiency of matching technology that can flexibly depend on the location pairs; and

$$\overline{M}_{ud,hl}^B = N_{u,h} \int n_{ud,hl}^B(z) dG_{u,h}(z) = N_{u,h} a_{ud,hl}^B \mathbb{M}_{u,h} \left(\frac{\delta_h}{\gamma_h^B} \right). \quad (\text{C.18})$$

$$\overline{M}_{ud,hl}^S = N_{d,l} \int n_{ud,hl}^S(z) dG_{d,l}(z) = N_{d,l} a_{ud,hl}^S \mathbb{M}_{d,l} \left(\frac{\delta_l}{\gamma_l^S} \right), \quad (\text{C.19})$$

are aggregate efforts for buyer and supplier link formation.

Given $M_{ud,hl}$, the matching rates m_{ud}^S and m_{ud}^B are defined by:

$$m_{ud,hl}^S = \frac{M_{ud,hl}}{\overline{M}_{ud,hl}^S}, \quad m_{ud,hl}^B = \frac{M_{ud,hl}}{\overline{M}_{ud,hl}^B}. \quad (\text{C.20})$$

C.2.3 Aggregate Trade Flows and Trade Elasticity

The analytical characterization of the firm relationship-formation decision, combined with the Cobb–Douglas matching technology, yields a tractable expression for the aggregate production

³Notice that λ_{hl}^B represents the elasticity with respect to the aggregate efforts for *buyer* link formation (by the suppliers), and λ_{hl}^S represents the elasticity with respect to aggregate efforts for *supplier* link formation (by the buyers).

networks and trade flows.

Lemma C.2. *The measure of supplier-to-buyer relationships from supplier location u to buyer location d (extensive margin), $M_{ud,hl}$, and the average transaction volume per relationship (intensive margin), $\bar{r}_{ud,hl}$, are given by the following gravity equations:*

$$M_{ud,hl} = \rho_{hl}^E \chi_{ud,hl}^E \zeta_{u,hl}^E \xi_{d,hl}^E, \quad \bar{r}_{ud,hl} = \varrho_{hl}^I \chi_{ud,hl}^I \zeta_{u,h}^I \xi_{d,hl}^I, \quad (\text{C.21})$$

where we define $\tilde{\lambda}_{hl}^S \equiv \lambda_{hl}^S / \gamma_l^S$, $\tilde{\lambda}_{hl}^B \equiv \lambda_{hl}^B / \gamma_h^B$, and

$$\begin{aligned} \rho_{hl}^E &= \left(\varrho_{hl}^I \right)^{\frac{\tilde{\lambda}_{hl}^S + \tilde{\lambda}_{hl}^B}{1 - \tilde{\lambda}_{hl}^S - \tilde{\lambda}_{hl}^B} (1 - \sigma_h)} \\ \zeta_{u,hl}^E &= C_{u,h}^{\frac{\tilde{\lambda}_{hl}^S + \tilde{\lambda}_{hl}^B}{1 - \tilde{\lambda}_{hl}^S - \tilde{\lambda}_{hl}^B} (1 - \sigma_h)} e_{u,h}^{-\frac{\tilde{\lambda}_{hl}^B}{1 - \tilde{\lambda}_{hl}^S - \tilde{\lambda}_{hl}^B}} \\ \xi_{d,hl}^E &= \left(\frac{R_{d,l}}{E_{d,l}} \right)^{\frac{\tilde{\lambda}_{hl}^B}{1 - \tilde{\lambda}_{hl}^S - \tilde{\lambda}_{hl}^B}} D_{d,hl}^{\frac{\tilde{\lambda}_{hl}^S + \tilde{\lambda}_{hl}^B}{1 - \tilde{\lambda}_{hl}^S - \tilde{\lambda}_{hl}^B}} e_{d,l}^{-\frac{\tilde{\lambda}_{hl}^S}{1 - \tilde{\lambda}_{hl}^S - \tilde{\lambda}_{hl}^B}} \\ \chi_{ud,hl}^E &= \tau_{ud,hl}^{\frac{\tilde{\lambda}_{hl}^S + \tilde{\lambda}_{hl}^B}{1 - \tilde{\lambda}_{hl}^S - \tilde{\lambda}_{hl}^B} (1 - \sigma_h)} \\ &\quad \times \left[\frac{\kappa_{ud,hl}}{(f_{ud,hl}^S)^{\tilde{\lambda}_{hl}^S} (f_{ud,hl}^B)^{\tilde{\lambda}_{hl}^B}} \left(N_{d,l} \mathbb{M}_{d,l} \left(\frac{\delta_l}{\gamma_l^S} \right) \right)^{\lambda_{hl}^S} \left(N_{u,h} \mathbb{M}_{u,h} \left(\frac{\delta_h}{\gamma_h^B} \right) \right)^{\lambda_{hl}^B} (\Gamma_{d,h}^S)^{\tilde{\lambda}_{hl}^S} (\Gamma_{u,l}^B)^{\tilde{\lambda}_{hl}^B} \right]^{\frac{1}{1 - \tilde{\lambda}_{hl}^S - \tilde{\lambda}_{hl}^B}} \end{aligned}$$

and

$$\begin{aligned} \varrho_{hl}^I &= \tilde{\sigma}_h^{1 - \sigma_h} \\ \chi_{ud,hl}^I &= (\tau_{ud,hl})^{1 - \sigma_h} \\ \zeta_{u,h}^I &= (C_{u,h})^{1 - \sigma_h} \\ \xi_{d,l}^I &= D_{d,hl} \end{aligned}$$

We define trade elasticity, defined by the partial derivative of $X_{ud,hl}$ with respect to iceberg cost $\tau_{ud,hl}$, fixing factor and intermediate goods prices as given:

$$\varepsilon_{hl} \equiv \frac{\sigma_h - 1}{1 - \tilde{\lambda}_{hl}^S - \tilde{\lambda}_{hl}^B} \quad (\text{C.22})$$

C.3 Market Clearing and Equilibrium

To close the model, we introduce the various market clearing conditions. Intermediate goods for each firm ω are used as inputs for production by other firms and local retailers:

$$q_{i,k}^R(\omega) + \sum_d \sum_l \int_{\psi: \omega \in S_{id}(\psi)} q_{id,kl}(\omega, \psi) \tau_{id,kl} d\psi = q_{i,k}(\omega). \quad (\text{C.23})$$

Relationship-formation efforts use local labor and retail goods:

$$\int_{\omega \in \Omega_{i,k}(\omega)} \left(\sum_{d,l} n_{id,kl}^B(\omega) + \sum_{u,h} n_{ui,hk}^S(\omega) \right) d\omega = \left(\frac{L_{i,k}^A}{\mu_k} \right)^{\mu_k} \left(\frac{Q_{i,k}^A}{1 - \mu_k} \right)^{1 - \mu_k}. \quad (\text{C.24})$$

Retail goods are used by final consumers and for relationship-formation efforts:

$$L_i Q_{i,k}^F + Q_{i,k}^A = Q_{i,k}. \quad (\text{C.25})$$

Labor is used for intermediate goods production and for relationship-formation efforts:

$$\sum_k \left(\int_{\omega \in \Omega_{i,k}(\omega)} l_{i,k}(\omega) d\omega + L_{i,k}^A \right) = L_i. \quad (\text{C.26})$$

The equilibrium is defined as follows:

Definition C.1. The equilibrium is defined by the set of prices $\{p_{id,kl}(\omega), P_{i,k}, e_{i,k}, w_i, C_{i,k}\}$ and quantities $\{q_{id,kl}(\omega, \psi), q_{i,k}^R(\omega), Q_{i,k}^F, Q_{i,k}^A, n_{id,kl}^B(\omega), n_{ui,hk}^S(\omega), A_{i,k}, l_{i,k}(\omega), L_{i,k}^A, \tilde{\pi}_{i,k}(\omega)\}$ that satisfy the following set of conditions:

1. Households maximize consumption given the budget constraint (C.7) with firm profit given by $\Pi_{i,k} = N_{i,k} \int_{\omega \in \Omega_{i,k}(\omega)} \tilde{\pi}_{i,k}(\omega) d\omega$
2. Firms make optimal pricing and production decisions for intermediate goods (C.3), (C.4) and relationship-formation decisions (C.11); retailers make optimal optimal production decisions for retail goods (C.6)
3. All markets clear (C.23), (C.25), (C.24), and (C.26)

C.4 System of Equilibrium Equations

C.4.1 Baseline Equilibrium

- Trade flows

$$X_{ud,hl} = \varrho_{hl} \chi_{ud,hl} \zeta_{u,hl} \xi_{d,hl}$$

where

$$\begin{aligned} \rho_{hl} &= (\varrho_{hl}^I)^{-\varepsilon_{hl}} \\ \chi_{ud,hl} &= (\kappa_{ud,hl}^*)^{\frac{1}{1 - \bar{\lambda}_{hl}^S - \bar{\lambda}_{hl}^B}} (\tau_{ud,hl})^{-\varepsilon_{hl}} \\ \zeta_{u,hl} &= \frac{\mathbb{M}_{u,h}(\delta_h)}{\mathbb{M}_{u,h}\left(\frac{\delta_h}{\gamma_h^B}\right)} C_{u,h}^{-\varepsilon_h} e_{u,h}^{-\frac{\bar{\lambda}_{hl}^B}{1 - \bar{\lambda}_{hl}^S - \bar{\lambda}_{hl}^B}} \\ \xi_{d,hl} &= \left(\frac{R_{d,l}}{E_{d,l}} \right)^{\frac{\bar{\lambda}_{hl}^S}{1 - \bar{\lambda}_{hl}^S - \bar{\lambda}_{hl}^B}} D_{d,hl}^{\frac{1}{1 - \bar{\lambda}_{hl}^S - \bar{\lambda}_{hl}^B}} e_{d,l}^{-\frac{\bar{\lambda}_{hl}^S}{1 - \bar{\lambda}_{hl}^S - \bar{\lambda}_{hl}^B}} \end{aligned}$$

- Intensive and extensive margin

$$\bar{r}_{ud,kl} = \varrho_k^I (\tau_{ud,kl})^{1-\sigma_k} \left(\frac{\mathbb{M}_{u,k}(\delta_k)}{\mathbb{M}_{u,k}\left(\frac{\delta_k}{\gamma_k^B}\right)} C_{u,k}^{1-\sigma_k} \right) D_{d,kl}$$

$$M_{ud,kl} = \frac{X_{ud,hl}}{\bar{r}_{ud,kl}}$$

- Revenue and expenditure

$$R_{i,k} = \sum_d \sum_l X_{id,kl} \quad (\text{C.27})$$

$$E_{i,k} = \sum_u \sum_h X_{ui,hk} \quad (\text{C.28})$$

- Cost shifter

$$(C'_{i,k})^{1-\sigma_k} = w_i^{\beta_{k,L}(1-\sigma_k)} \times \prod_h \left(\left(N_{i,k} \mathbb{M}_{i,k} \left(\frac{\delta_k}{\gamma_k^S} \right) \right)^{-1} \frac{\tilde{\sigma}_h^{1-\sigma_h} \mathbb{M}_{u,h}(\delta_h)}{\mathbb{M}_{u,h}\left(\frac{\delta_h}{\gamma_h^B}\right)} \sum_{u \in \mathcal{N}} M_{ui,hk} C_{u,h}^{1-\sigma_h} \tau_{ui,hk}^{1-\sigma_h} \right)^{\frac{\beta_{hk}}{1-\sigma_h}(1-\sigma_k)} \quad (\text{C.29})$$

- Labor compensation

$$w_i L_i = \sum_k \underbrace{\frac{\beta_{k,L}}{\sum_h \beta_{hk}} E_{i,k}}_{\text{production}} + \underbrace{\frac{1}{\sigma_k} \mu_k \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S} \right) R_{i,k}}_{\text{relationship formation}} \quad (\text{C.30})$$

- Profit

$$\Pi_i = \sum_k \frac{1}{\sigma_k} \left(1 - \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S} \right) \right) R_{i,k} \quad (\text{C.31})$$

- Relationship-formation cost

$$e_{i,k} = (w_i)^{\mu_k} (P_{i,k})^{1-\mu_k}$$

- Aggregate expenditure for sector k : by noting that there is no profit from sales to retailers,

$$\begin{aligned} E_{i,k} &= \left(\sum_h \beta_{hk} \right) \left[P_{i,k} Q_{i,k}^F + P_{i,k} Q_{i,k}^A + \frac{\sigma_k-1}{\sigma_k} R_{i,k} \right] \\ &= \left(\sum_h \beta_{hk} \right) \left[\alpha_k (w_i L_i + \Pi_i) + \left[\frac{1}{\sigma_k} (1 - \mu_k) \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S} \right) + \frac{\sigma_k-1}{\sigma_k} \right] R_{i,k} \right] \end{aligned} \quad (\text{C.32})$$

C.4.2 Counterfactual Equilibrium

- Trade flows

$$\hat{X}_{ud,hl} = \hat{\chi}_{ud,hl} \hat{\zeta}_{u,hl} \hat{\xi}_{d,hl}, \quad \hat{\zeta}_{u,hl} = \hat{C}_{u,h}^{-\varepsilon_{hl}} \hat{e}_{u,h}^{-\frac{\tilde{\lambda}_{hl}^B}{1-\tilde{\lambda}_{hl}^S-\tilde{\lambda}_{hl}^B}}, \quad \hat{\xi}_{d,hl} = \left(\frac{\hat{R}_{d,l}}{\hat{E}_{d,l}} \right)^{\frac{\tilde{\lambda}_{hl}^S}{1-\tilde{\lambda}_{hl}^S-\tilde{\lambda}_{hl}^B}} \hat{D}_{d,hl}^{\frac{1}{1-\tilde{\lambda}_{hl}^S-\tilde{\lambda}_{hl}^B}} \hat{e}_{d,l}^{-\frac{\tilde{\lambda}_{hl}^S}{1-\tilde{\lambda}_{hl}^S-\tilde{\lambda}_{hl}^B}}$$

- Intensive and extensive margin

$$\hat{M}_{ui,hk} = \frac{\hat{X}_{ud,hl}}{\hat{T}_{ud,kl}}, \quad \hat{r}_{ud,kl} = (\hat{\tau}_{ud,kl})^{1-\sigma_k} \hat{C}_{u,k}^{1-\sigma_k} \hat{D}_{d,l}$$

- Revenue

$$\hat{R}_{i,k} = \sum_d \sum_l \Psi_{id,kl} \hat{X}_{id,kl}$$

- Cost shifter

$$\left(\hat{C}_{i,k} \right)^{1-\sigma_k} = \hat{w}_i^{\beta_{k,L}(1-\sigma_k)} \prod_h \left(\sum_{u \in \mathcal{N}} \hat{M}_{ui,hk} \left(\hat{C}_{u,h} \right)^{1-\sigma_h} (\hat{\tau}_{ui,hk})^{1-\sigma_h} \Lambda_{ui,hk} \right)^{\frac{\beta_{hk}}{1-\sigma_h}(1-\sigma_k)}$$

- Labor compensation

$$\hat{w}_i = \sum_k S_{i,k}^{L,F} \hat{E}_{i,k} + S_{i,k}^{L,A} \hat{R}_{i,k}$$

- Profit

$$\hat{\Pi}_i = \sum_k S_{i,k}^{\Pi} \hat{R}_{i,k}$$

- Relationship-formation cost

$$\hat{e}_{i,k} = (\hat{w}_i)^{\mu_k} \left(\hat{C}_{i,k} \right)^{1-\mu_k}$$

- Expenditure

$$\hat{E}_{i,k} = S_{i,k}^{E,L} \hat{w}_i + S_{i,k}^{E,\Pi} \hat{\Pi}_i + S_{i,k}^{E,R} \hat{R}_{i,k}$$

- Demand shifter

$$\hat{D}_{d,hl} = \hat{e}_{d,l}^{\tilde{\lambda}_{hl}^B} \left(\frac{\hat{R}_{d,l}}{\hat{E}_{d,l}} \right)^{-\tilde{\lambda}_{hl}^S} \left(\widehat{\sum_u X_{ud,hl}} \right)^{1-\tilde{\lambda}_{hl}^S-\tilde{\lambda}_{hl}^B} \left(\sum_{u,h} \Lambda_{ui,hk} \hat{\chi}_{ud,hl} \hat{C}_{u,h}^{-\varepsilon_{hl}} \hat{e}_{u,h}^{-\frac{\tilde{\lambda}_{hl}^S}{1-\tilde{\lambda}_{hl}^S-\tilde{\lambda}_{hl}^B}} \right)^{-(1-\tilde{\lambda}_{hl}^S-\tilde{\lambda}_{hl}^B)}$$

where the baseline shares are given by

- Revenue share

$$\Psi_{id,kl} = \frac{X_{id,kl}}{\sum_{d'} \sum_{l'} X_{id',kl'}}$$

- Expenditure share

$$\Lambda_{ui,hk} = \frac{X_{ui,hk}}{\sum_{u'} X_{u'i,h'k}}$$

- Labor compensation shares across sectors

$$S_{i,k}^{L,F} = \frac{\frac{\beta_{k,L}}{\sum \beta_{hk}} E_{i,k}}{\sum_k \frac{\beta_{k,L}}{\sum \beta_{hk}} E_{i,k} + \frac{1}{\sigma_k} \mu_k \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S} \right) R_{i,k}}$$

$$S_{i,k}^{L,A} = \frac{\frac{1}{\sigma_k} \mu_k \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S} \right) R_{i,k}}{\sum_k \frac{\beta_{k,L}}{\sum \beta_{hk}} E_{i,k} + \frac{1}{\sigma_k} \mu_k \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S} \right) R_{i,k}}$$

- Profit share across sectors

$$S_{i,k}^{\Pi} = \frac{\frac{1}{\sigma_k} \left(1 - \mu_k \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S} \right) \right) R_{i,k}}{\sum_k \frac{1}{\sigma_k} \left(1 - \mu_k \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S} \right) \right) R_{i,k}}$$

- Demand shares

$$S_{i,k}^{E,L} = \frac{\alpha_k w_i L_i}{\alpha_k (w_i L_i + \Pi_i) + \left[\frac{1}{\sigma_k} (1 - \mu_k) \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S} \right) + \frac{\sigma_k-1}{\sigma_k} \right] R_{i,k}}$$

$$S_{i,k}^{E,\Pi} = \frac{\alpha_k \Pi_i}{\alpha_k (w_i L_i + \Pi_i) + \left[\frac{1}{\sigma_k} (1 - \mu_k) \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S} \right) + \frac{\sigma_k-1}{\sigma_k} \right] R_{i,k}}$$

$$S_{i,k}^{E,R} = \frac{\left[\frac{1}{\sigma_k} (1 - \mu_k) \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S} \right) + \frac{\sigma_k-1}{\sigma_k} \right] R_{i,k}}{\alpha_k (w_i L_i + \Pi_i) + \left[\frac{1}{\sigma_k} (1 - \mu_k) \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S} \right) + \frac{\sigma_k-1}{\sigma_k} \right] R_{i,k}}$$

where

$$w_i L_i = \underbrace{\sum_k \frac{\beta_{k,L}}{\sum \beta_{hk}} E_{i,k}}_{\text{production}} + \underbrace{\frac{1}{\sigma_k} \mu_k \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S} \right) R_{i,k}}_{\text{relationship formation}}$$

$$\Pi_i = \sum_k \frac{1}{\sigma_k} \left(1 - \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S} \right) \right) R_{i,k}$$

C.5 Planning Problem

We consider a planner with a restrictive set of policy tools. First, the planner can introduce linear (ad-valorem) subsidies for intermediate goods sales specific to origin i and destination d , $s_{id,kl}^I$. Under these subsidies, the intermediate goods prices change from equation (4) to

$$p_{id,kl}(\nu, \omega) = (1 - s_{id,kl}^I) \tilde{\sigma}_k c_{i,k}(\nu) \tau_{id,kl}. \quad (\text{C.33})$$

Second, the planner can introduce linear taxes for supplier and buyer relationship-formation effort, $t_{id,kl}^S$ and $t_{id,kl}^B$ for each pair of supplier and buyer locations. Therefore, the total relationship-formation costs by firms in location i are modified as

$$\begin{aligned} & f_{i,k}(\{n_{id,kl}^B\}_{d,l}, \{n_{ui,mk}^S\}_{u,m}) \\ &= e_{i,k} \left\{ \sum_{d \in \mathcal{N}} \sum_l (1 + t_{id,kl}^B) f_{id,kl}^B \frac{(n_{id,kl}^B)^{\gamma_k^B}}{\gamma_k^B} + \sum_{u \in \mathcal{N}} \sum_h (1 + t_{ui,hk}^S) f_{ui,hk}^S \frac{(n_{ui,hk}^S)^{\gamma_k^S}}{\gamma_k^S} \right\}, \quad (\text{C.34}) \end{aligned}$$

Finally, we introduce lump-sum transfers for households in location i , T_i^F , so that households' budget constraint is modified from equation (7) to $\sum_k P_{i,k} Q_{i,k}^F = w_i + \frac{\Pi_i}{L_i} + T_i^F$.

The planner chooses the optimal policy to maximize the global welfare

$$\max_{\{s_{id,ml}^I, t_{id,ml}^B, t_{id,ml}^S\}, T_i^F} \mathcal{W} \equiv \sum_i \psi_i L_i \prod_k (Q_{i,k}^F)^{\alpha_k} \quad (\text{C.35})$$

subject to equilibrium constraints and the government budget constraint, where $\psi_i \geq 0$ corresponds to the welfare weights attached to each location. The following proposition provides a simple formula for the optimal policy.

Proposition C.1. *The optimal set of taxes and subsidies $\{s_{id}^I, t_{id}^B, t_{id}^S\}$ must satisfy*

$$s_{id,ml}^I = \frac{1}{\sigma_m}, \quad t_{id,ml}^B = \frac{1}{\lambda_{ml}^B} - 1, \quad t_{id,ml}^S = \frac{1}{\lambda_{ml}^S} \frac{\sigma_m - 1}{\sigma_m} \frac{\sigma_l}{\sigma_l - 1} \beta_{ml} \frac{E_{d,l}}{R_{d,l}} - 1, \quad (\text{C.36})$$

for all i, d , where $R_{i,k} = \sum_{d,l} X_{id,kl}$ corresponds to aggregate intermediate goods sales (excluding those to retailers) in location d and $E_{i,k} = \sum_{u,h} X_{ui,hk}$ corresponds to the aggregate intermediate goods expenditure in location d . Lump-sum transfers $\{T_i^F\}$ is set depending on the welfare weights ψ_i .

Proof. Similarly as Proposition 2, we solve for a relaxed planning problem where the planner

directly specifies the allocation. The relaxed problem is given by

$$\begin{aligned}
& \max_{\{Q_{i,k}^F, Q_{i,k}^A, L_{i,k}^A, L_{i,k}^P, A_{ui,hk}^S, A_{ui,hk}^B, q_{i,k}^R, q_{id,hk}, M_{ui,hk}\}} \sum_i \psi_i L_i \prod_k (Q_{i,k}^F)^{\alpha_k} & (C.37) \\
& \text{s.t. } Q_{i,k}^A + L_i Q_{i,k}^F = g_{i,k} (q_{i,k}^R) & [P_{i,k}] \\
& \sum_k (L_{i,k}^A + L_{i,k}^P) = L_i & [w_i] \\
& \sum_h \sum_u A_{ui,hk}^S + \sum_l \sum_d A_{id,kl}^B = \left(\frac{L_{i,k}^A}{\mu_k} \right)^{\mu_k} \left(\frac{Q_{i,k}^A}{1 - \mu_k} \right)^{1 - \mu_k} & [e_{i,k}] \\
& q_{i,k}^R + \sum_m \sum_d q_{id,km} \tau_{id,km} = Z_{i,k} \left(\frac{L_{i,k}^P}{\beta_{k,L}} \right)^{\beta_{k,L}} \prod_h \left(\frac{\left(\sum_u M_{ui,hk}^{\frac{1}{\sigma_h}} q_{ui,hk}^{\frac{\sigma_h-1}{\sigma_h}} \right)^{\frac{\sigma_h}{\sigma_h-1}}}{\beta_{hk}} \right)^{\beta_{hk}} & [C_{i,k}] \\
& M_{ui,hk} = \kappa_{ui,hk} \left(\frac{\gamma_h^B}{f_{ui,hk}^B} A_{ui,hk}^B \right)^{\frac{\lambda_{hk}^B}{\gamma_h^B}} \left(\frac{\gamma_k^S}{f_{ui,hk}^S} A_{ui,hk}^S \right)^{\frac{\lambda_{hk}^S}{\gamma_k^S}} & [\eta_{ui,hk}]
\end{aligned}$$

The FOCs of this problem with respect to $Q_{i,k}^F$, $Q_{i,k}^A$, $q_{i,k}^R$, $L_{i,k}^A$, $L_{i,k}^P$, $A_{ui,hk}^B$, $A_{ui,hk}^S$, $M_{ui,hk}$, $q_{ui,hk}$ is given by:

$$P_i = \psi_i L_i \alpha_k \frac{W_i}{Q_{i,k}^F} \quad (C.38)$$

$$P_{i,k} = e_{i,k} A_{i,k} (1 - \mu_k) \frac{1}{Q_{i,k}^A} \quad (C.39)$$

$$C_{i,k} = P_{i,k} g'_{i,k} (q_{i,k}^R) \quad (C.40)$$

$$w_i = e_{i,k} A_{i,k} \mu_k \frac{1}{L_{i,k}^A} \quad (C.41)$$

$$w_i = \beta_k C_{i,k} q_{i,k} \frac{1}{L_{i,k}^P} \quad (C.42)$$

$$e_{u,h} = \eta_{ui,hk} \frac{\lambda_{hk}^B}{\gamma_h^B} M_{ui,hk} \frac{1}{A_{ui,hk}^B} \quad (C.43)$$

$$e_{i,k} = \eta_{ui,hk} \frac{\lambda_{hk}^S}{\gamma_k^S} M_{ui,hk} \frac{1}{A_{ui,hk}^S} \quad (C.44)$$

$$\eta_{ui,hk} = \frac{1}{\sigma_h - 1} \beta_{hk} C_{i,k} q_{i,k} \Lambda_{ui,hk} \frac{1}{M_{ui,hk}} \quad (C.45)$$

$$\tau_{ui,hk} C_{u,h} = \beta_{hk} C_{i,k} q_{i,k} \Lambda_{ui,hk} \frac{1}{q_{ui,hk}} \quad (C.46)$$

where $W_i = \prod_k (Q_{i,k}^F)^{\alpha_k}$, $A_{i,k} \equiv \sum_h \sum_u A_{ui,hk}^S + \sum_l \sum_d A_{id,kl}^B$, $q_{i,k} \equiv q_{i,k}^R + \sum_m \sum_d q_{id,km} \tau_{id,km}$,

and $\Lambda_{ui,hk} \equiv \frac{M_{ui,hk}^{\frac{1}{\sigma_h}} q_{ui,hk}^{\frac{\sigma_h-1}{\sigma_h}}}{\sum_{\ell} M_{\ell i,hk}^{\frac{1}{\sigma_h}} q_{\ell i,hk}^{\frac{\sigma_h-1}{\sigma_h}}}$. From (C.43), (C.44), (C.45),

$$\frac{\lambda_{hk}^B}{\gamma_h^B} \frac{1}{\sigma_h - 1} \beta_{hk} C_{i,k} q_{i,k} \Lambda_{ui,hk} = e_{u,h} A_{ui,hk}^B \quad (\text{C.47})$$

$$\frac{\lambda_{hk}^S}{\gamma_k^S} \frac{1}{\sigma_k - 1} \beta_{hk} C_{i,k} q_{i,k} \Lambda_{ui,hk} = e_{i,k} A_{ui,hk}^S \quad (\text{C.48})$$

We now consider the equilibrium subsidies/taxes that implement this allocation. Suppose that the Lagrange multipliers $\{P_{i,k}, w_i, e_{i,k}, C_{i,k}\}$ correspond to equilibrium prices. Then, from the intermediate goods market clearing condition,

$$(1 - s_{ui,hk}^I) \frac{\sigma_h}{\sigma_h - 1} \tau_{ui,hk} C_{u,h} q_{ui,hk} = \beta_{hk} C_{i,k} q_{i,k} \Lambda_{ui,hk} \quad (\text{C.49})$$

By comparing this condition with (C.46), the optimal $s_{ui,hk}^I$ satisfies $1 - s_{ui,hk}^I = 1 - s_h^I = \frac{\sigma_h-1}{\sigma_h} \iff s_h^I = 1/\sigma_h$.

Next, following Lemma C.1 applied to the economy with taxes, the equilibrium allocation for buyer and supplier relationship formation is given by

$$(1 + t_{ui,hk}^B) e_{u,h} A_{ui,hk}^B = \frac{1}{\sigma_h} \frac{1}{\gamma_h^B} X_{ui,hk} \frac{1}{1 - s_h^I} \quad (\text{C.50})$$

$$(1 + t_{ui,hk}^S) e_{i,k} A_{ui,hk}^S = \frac{1}{\sigma_k} \frac{\beta_{hk}}{\gamma_k^S} X_{ui,hk} \frac{R_{i,k}}{E_{i,k}} \frac{1}{1 - s_h^I} \quad (\text{C.51})$$

By comparing these equations with (C.47) and (C.48), and by noting that $X_{ui,hk} = \beta_{hk} C_{i,k} q_{i,k} \Lambda_{ui,hk}$, we obtain the expressions in Proposition C.1.

Finally, we confirm that these taxes can be implemented in the equilibrium. Notice that the constraints in (C.37) coincide with resource constraints, relationship-formation effort, and matching technology; (C.42) and (C.46) coincide with intermediate goods producers' optimal production decisions; (C.40) coincides with retailers' optimal production decisions; (C.39), (A.31), (C.50) and (C.51) coincide with intermediate producers' optimal relationship-formation decisions. Finally, lump-sum transfers can be set such that

$$T_i^F = \sum_k P_{i,k} Q_{i,k}^F - \frac{\sum_k \Pi_{i,k}^F}{L_i} - w_i$$

We can verify that these allocations satisfy the government budget constraints from market-clearing conditions. $\square$

C.6 Ex-Post Sufficient Statistics on Each Location's Welfare

The welfare gains from trade from an external shock to location i are characterized by the following proposition:

Proposition C.2. *The welfare changes in location i from external shocks is given by $d \ln W_i = -\sum_h \alpha_h d \ln C_{i,h}$, where*

$$d \ln C_{i,h} = \sum_k \phi_{hk} \sum_l \beta_{lh} \left(\frac{1}{\varepsilon_{lh}} d \ln \Lambda_{ii,lh} + \frac{1}{\sigma_l - 1} \left(\tilde{\lambda}_{lh}^S d \ln R_{i,h} + \tilde{\lambda}_{lh}^B d \ln E_{i,h} \right) \right) \quad (\text{C.52})$$

where $\Lambda_{ii,kl} = X_{ii,kl} / \sum_u X_{ui,kl}$, and ϕ_{kh} is an (k, h) -th element of a matrix Φ given by

$$\Phi = [\mathbf{c}^S \mathbf{I} - \mathbf{C}^B]^{-1} \quad (\text{C.53})$$

where $\mathbf{I}$ is the $|K| \times |K|$ identity matrix, $\mathbf{c}^S$ is the $|K| \times 1$ vector with k -th element corresponds to $1 - \sum_h \beta_{hk} \frac{1-\mu_k}{\sigma_h-1} \tilde{\lambda}_{hk}^B$, and $\mathbf{C}^B$ is the $|K| \times |K|$ matrix with (k, h) -th element corresponds to $\beta_{hk} \left(1 + \frac{1-\mu_h}{\sigma_h-1} \tilde{\lambda}_{hk}^S \right)$.

Note that Proposition C.2 collapses to Proposition 4 in the case of a single sector. Note that, in the case of a single sector, $d \ln R_i = d \ln E_i = d \ln w_i$ because of the trade balance and constant share of intermediate goods sales to labor, and hence $d \ln \Lambda_{ii}$ is the only endogenous variable summarizing the welfare gains from trade.

Proof. We start by applying Shephard's Lemma to the firm's cost minimization problem (C.14). For expositional simplicity, we only consider shocks to external trade costs $\{d \ln \tau_{ui}\}$, while the final equation (C.54) is unchanged for any other external shocks considered in Proposition C.2.

Applying the Shephard's Lemma to the firm's cost minimization problem (C.14),

$$\begin{aligned} d \ln C_{i,k} &= \sum_h \beta_{hk} \sum_u \Lambda_{ui,hk} (d \ln C_{u,h} + d \ln \tau_{ui,hk} - \frac{1}{\sigma_h - 1} d \ln M_{ui,hk}) \\ &= \sum_h \beta_{hk} (d \ln C_{i,h} - \frac{1}{\sigma_h - 1} d \ln M_{ii,hk} + \frac{1}{\sigma_h - 1} d \ln \Lambda_{ii,hk}) \end{aligned} \quad (\text{C.54})$$

At the same time, from Lemmas C.1 and C.2, and from the cost minimization of retailers,

$$\begin{aligned} d \ln M_{ii,hk} &= \left(\tilde{\lambda}_{hk}^S + \tilde{\lambda}_{hk}^B \right) \left(\underbrace{d \ln X_{ii,hk}}_{=d \ln \Lambda_{ii,hk} + d \ln E_{i,k}} \right) + \tilde{\lambda}_{hk}^S (d \ln R_{i,k} - d \ln E_{i,k}) \\ &\quad - \left((1 - \mu_h) \tilde{\lambda}_{hk}^B d \ln C_{i,h} + (1 - \mu_k) \tilde{\lambda}_{hk}^S d \ln C_{i,k} \right) \\ &= \left(\tilde{\lambda}_{hk}^S + \tilde{\lambda}_{hk}^B \right) d \ln \Lambda_{ii,hk} + \tilde{\lambda}_{hk}^S d \ln R_{i,k} + \tilde{\lambda}_{hk}^B d \ln E_{i,k} \\ &\quad - \left((1 - \mu_h) \tilde{\lambda}_{hk}^B d \ln C_{i,h} + (1 - \mu_k) \tilde{\lambda}_{hk}^S d \ln C_{i,k} \right) \end{aligned}$$

where $R_{i,k}$ and $E_{i,k}$ denote location i and sector k 's intermediate goods sales and expenditure. Note that, unlike the case for a single sector, these two objects may not take the same value.

Combining this expression with equation (C.54),

$$\begin{aligned} & \left(1 - \sum_h \beta_{hk} \frac{1 - \mu_k}{\sigma_h - 1} \tilde{\lambda}_{hk}^B\right) d \ln C_{i,k} \\ &= \sum_h \beta_{hk} \left(1 + \frac{1 - \mu_h}{\sigma_h - 1} \tilde{\lambda}_{hk}^S\right) d \ln C_{i,h} + \frac{\beta_{hk}}{\varepsilon_{hk}} d \ln \Lambda_{ii,hk} + \frac{\beta_{hk}}{\sigma_h - 1} \left(\tilde{\lambda}_{hk}^S d \ln R_{i,k} + \tilde{\lambda}_{hk}^B d \ln E_{i,k}\right) \end{aligned}$$

Therefore, denoting $\mathbf{c}^S$ as $|K| \times 1$ vector with k -th element corresponds to $1 - \sum_h \beta_{hk} \frac{1 - \mu_k}{\sigma_h - 1} \tilde{\lambda}_{hk}^B$, and $\mathbf{C}^B$ as $|K| \times |K|$ matrix with (k, h) -th element corresponds to $\beta_{hk} \left(1 + \frac{1 - \mu_h}{\sigma_h - 1} \tilde{\lambda}_{hk}^S\right)$. Then,

$$d \ln C_{i,k} = \sum_h \phi_{kh} \sum_l \beta_{lh} \left(\frac{1}{\varepsilon_{lh}} d \ln \Lambda_{ii,lh} + \frac{1}{\sigma_l - 1} \left(\tilde{\lambda}_{lh}^S d \ln R_{i,h} + \tilde{\lambda}_{lh}^B d \ln E_{i,h} \right) \right)$$

where ϕ_{kh} is an (k, h) -th element of a matrix Φ given by

$$\Phi = [\mathbf{c}^{S'} \mathbf{I} - \mathbf{C}^B]^{-1}$$

where $\mathbf{I}$ is the $|K| \times |K|$ identity matrix. Finally, welfare gains from trade in location i is given by

$$d \ln W_i = - \sum_k \alpha_k d \ln C_{i,k}.$$

□

Now we derive the expression for aggregate intermediate goods sales (excluding those to retailers), $R_{i,h}$, and aggregate intermediate goods expenditure, $E_{i,h}$, under regional autarky, similarly as in [Costinot and Rodríguez-Clare \(2014\)](#). Note the normalization of region i 's GDP to one such that $w_i L_i + \Pi_i = 1$. First, note that

$$R_{i,k} = \sum_h \frac{\beta_{kh}}{\sum_l \beta_{lh}} E_{i,h}$$

Furthermore,

$$\begin{aligned} E_{i,k} &= \left(\sum_l \beta_{lk} \right) \left[P_{i,k} Q_{i,k}^F + P_{i,k} Q_{i,k}^A + \frac{\sigma_k - 1}{\sigma_k} R_{i,k} \right] \\ &= \left(\sum_l \beta_{lk} \right) \left[\underbrace{\alpha_k (w_i L_i + \Pi_i)}_{=1} + \left[\frac{1}{\sigma_k} (1 - \mu_k) \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S} \right) + \frac{\sigma_k - 1}{\sigma_k} \right] R_{i,k} \right] \\ &= \left(\sum_l \beta_{lk} \right) \left[\alpha_k + \left[\frac{1}{\sigma_k} (1 - \mu_k) \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S} \right) + \frac{\sigma_k - 1}{\sigma_k} \right] \sum_h \frac{\beta_{kh}}{\sum_l \beta_{lh}} E_{i,h} \right] \end{aligned}$$

$$\tilde{E}_{i,k} = \left[\alpha_k + \left[\frac{1}{\sigma_k} (1 - \mu_k) \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S} \right) + \frac{\sigma_k - 1}{\sigma_k} \right] \sum_h \beta_{kh} \tilde{E}_{i,h} \right]$$

where we define $\tilde{E}_{i,k} \equiv E_{i,k} / (\sum_l \beta_{lk})$. Then,

$$\tilde{\mathbf{E}}_i = \tilde{\mathbf{E}} = [\mathbf{I} - \mathbf{D}]^{-1} \mathbf{a}$$

where $\tilde{\mathbf{E}}$ is $|K| \times 1$ vector with k 'th element correspond to $\tilde{E}_{i,k}$, $\mathbf{D}$ is $|K| \times |K|$ matrix with (k, h) -th element corresponds to $\beta_{kh} \left[\frac{1}{\sigma_k} (1 - \mu_k) \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S} \right) + \frac{\sigma_k - 1}{\sigma_k} \right]$, and $\mathbf{a}$ is $|K| \times 1$ vector of α_k .

Once we obtain $E_{i,k}$, we can further back out $R_{i,k} = \sum_h \frac{\beta_{kh}}{\sum_l \beta_{lh}} E_{i,h}$.

D Appendix for Data and Reduced-Form Facts

D.1 Empirical Estimates of Gravity Equations

This appendix presents the estimates of the gravity equations of domestic firm-to-firm trade flows across Chilean municipalities. In Table D.1, we document that geographic frictions shape the architecture of production networks. Specifically, we construct the total trade flows, average transaction per relationship (intensive margin), and the number of supplier-to-buyer relationships (extensive margin), across 345 municipalities within Chile, by aggregating our domestic firm-to-firm trade data in 2019. We then regress the log of these values on the log of travel distance between the municipalities, controlling for the origin and destination municipality fixed effects. These regressions correspond to the structural gravity equations implied by our model (Lemma 2), where we parameterize iceberg trade costs and matching frictions as an isoelastic function of travel distance.

We find that the overall trade flows sharply decline in geographic distance, with the estimated elasticity of -1.43 . This estimate is within the range but on the higher end of the typical estimates for international trade flows (0.28-1.55; [Disdier and Head, 2008](#)). Interestingly, this decay is driven both by the transaction volume per relationship (intensive margin), with the elasticity of -0.52 , and the number of supplier-to-buyer relationships (extensive margin), with the elasticity of -0.91 . This pattern is consistent with Lemma 2, which predicts that iceberg trade costs and matching frictions drive distinct spatial structures for extensive and intensive margins.

Table D.1: Estimates of Empirical Gravity Regression within Chile

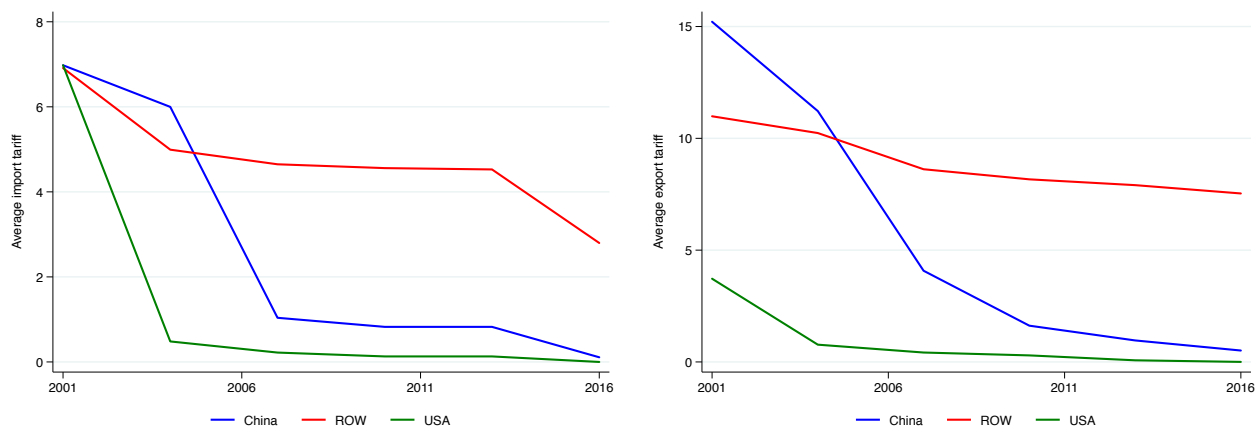
	Total	Intensive	Extensive
	(1)	(2)	(3)
Log Distance	-1.43 (0.01)	-0.52 (0.01)	-0.91 (0.00)
R^2	0.63	0.30	0.83
Origin Municipality FE	✓	✓	✓
Destination Municipality FE	✓	✓	✓
Same Municipality FE	✓	✓	✓
Number of Observations	72668	72668	72668

Notes: This table presents the results of estimating the gravity regressions, where we regress the logarithm of the total transaction volume between a pair of municipalities on the logarithm of travel distance, controlling for origin and destination fixed effects and the dummies for the same municipalities, across 345 municipalities within Chile, using our domestic firm-to-firm trade data in 2019. The dependent variable corresponds to log total trade flow, log average trade flow (intensive margin), and the log number of links between municipalities (extensive margin).

D.2 Tariff Changes from Bilateral Trade Agreements

The U.S. and Chile implemented a Preferential Trade Agreement (PTA) in 2004.⁴ This PTA reduced Chile's (average) preferential import tariff towards the U.S. products by 93% (from an average applied tariff of 6.9 percentage point to 0.5 percentage point (Fontagné et al., 2022)), with a peak of a 100% tariff cut (i.e. the complete removal of import tariffs) for many organic and inorganic chemical products as well as for many plastic and rubber products (Fontagné et al., 2022). It had similar effects on the export tariffs of Chile to the U.S. In addition, Chile has implemented a trade liberalization agenda that, in particular, reduced tariffs from and to China. Average import tariffs with China were reduced from 6.9 percentage points in 2001 to 0.1 percentage points in 2016. Figure D.1 shows a significant tariff decline from and to China and the U.S., while there is only a moderate decline to the Rest of the World (ROW). These tariff cuts were particularly relevant for intermediate imports, as one of the products that is most imported from China is computers and engines, whereas from the U.S. are energy-related inputs such as gas and chemical products. Table D.2 summarizes the tariff changes between 2001 and 2016 from and to the U.S. and China for three main sectors where the majority of trade liberalization occurred: Agricultural and Fishing, Mining, and Manufacturing. While import tariff reduction is relatively homogeneous across sectors, export tariff reduction is heterogeneous across sectors.

Figure D.1: Import and Export Tariffs of Chile during 2001-2016
(a) Average Import Tariffs (b) Average Export Tariffs



Notes: These figures present the average import and export tariffs of Chile (averaged across sectors) with China, the U.S., and the rest of the world (ROW), computed using the dataset built by Fontagné et al. (2022). Panel (A) presents average import tariffs, and Panel (B) presents average export tariffs (imposed by the counterpart countries).

D.3 Shift-Share Design

In this section, we discuss the validity of the shift-share design of our trade shocks, following Borusyak et al. (2022) (henceforth BHJ). The identification assumption of the shift-share design,

⁴In 2018, imports from China and the U.S. constituted about 24% and 19% of overall imports, which corresponds to 6% and 4% of Chile's GDP, respectively. Exports to China and the U.S. were about 33% and 14% of overall exports, which amounts to 8% and 4% of Chile's GDP, respectively.

Table D.2: Import and Export Tariff Change of Chile with Main Trade Partners: Across Sectors (%)

	Imports		Exports	
	China	US	China	US
a) Agriculture and Fishing	-6.54	-6.54	-12.84	-1.86
b) Mining	-6.45	-6.45	-2.63	-0.20
c) Manufacturing	-6.45	-6.45	-13.06	-3.85

Notes: This table presents the average percentage point changes in tariffs from and to China and the U.S., between 2001 and 2016, across different sectors and for import and export tariffs, computed using the dataset built by [Fontagné et al. \(2022\)](#).

as established in BHJ, relies on the exogeneity of the shifts, allowing exposure shares to be potentially endogenous. Concretely, the consistency of the shift-share instrumental variable (SSIV) estimator requires the shifts to be (i) randomly assigned, (ii) numerous and mutually uncorrelated, and (iii) relevant. We focus on this appendix in discussing conditions (ii) and (iii). In our context, shifts are defined by tariff changes between 2003 and 2007 at the country-product level. Shares of exposure are defined as import shares relative to total imports (equation 41) and relative to total expenses (equation 42).

We start with condition (ii). Table D.3 presents summary statistics of importance weights $s_{cp} = \sum_f s_{fcp}$ as in BHJ, where s_{fcp} is the share associated with the triple of firm f , seller country c , and HS-6 product p . As discussed in Section 6.2, we used two versions of import shocks: the weighted average of import tariff changes in the basket of imported goods as in equation (41) (using “Total Imports” as the denominator), and (2) the weighted average of import tariff changes in the basket of all sourced goods (from domestic and international suppliers) as in equation (42) (using “Total Expenses” as the denominator). Both shocks have similar means, standard deviations, and interquartile ranges. We rely on tariff variation across 113 countries, 4322 products, and 33260 country-product combinations. To assess whether shocks are not too concentrated, we compute their inverse HHI $1 / \sum_f s_{fcp}^2$ across all country-product pairs, which is coined as the effective sample size. Following BHJ, the asymptotics of this design rely on this measure being large. The effective sample size is indeed high, suggesting that condition ii) holds. We also see that the largest weight in our sample is below 0.01, so no single country-product pair is particularly important for firms in the sample. Following the discussion in BHJ, this suggests that a shock-level law of large numbers applies for these shocks.⁵

Table D.4 shows further statistics of the shock at the firm-level. We conclude three features of the shock. First, shocks have similar standard deviations. Second, adding sectoral fixed effects does not remove much of the variation in both shocks. Third, controlling for “incomplete shares” does play a role for the shock that uses total expenses as the denominator.⁶

Finally, we check whether our shocks are in fact relevant (condition iii). We follow [Demir](#)

⁵Table D.3 also shows that, once we compute importance weights at either the country region-product group or product-group levels, the effective sample size decreases substantially. This implies that there is much less variation available if we allow shocks to be correlated or clustered at this level.

⁶It does not play a role for the shock that uses total imports in the denominator as shares are complete in this case.

Table D.3: Shift-Share Import Shock: Summary Statistics

Share Denominator	Total Imports (1)	Total Expenses (2)
Mean	-0.05	-0.05
Standard deviation	0.03	0.03
Interquartile range	0.04	0.04
Effective sample size		
Across country-products	2453	2324
Across region-HS3	187	187
Across HS3	61	61
Largest weight		
Across country-products	0.00	0.00
Across region-HS3	0.02	0.02
Across HS3	0.04	0.04
Number of countries	113	113
Number of products	4322	4322
Number of region-HS3	869	869
Number of HS3	217	217
Number of shocks (country-products)	33260	33260

Notes: This table presents summary statistics of the shift-share import shock used for the empirical and quantitative analysis. Column 1 uses the definition of the shock where the denominator of the share is total imports as described in equation (41). Column 2 uses the definition of the shock where the denominator of the share is total expenses as described in equation (42).

Table D.4: Shift-Share Import Shock: Dispersion

Shock Denominator	Controls	Mean	SD	IQR
(1) Total Imports	Raw	-0.052	0.019	0.027
	Net of FE	0.000	0.019	0.024
	Net of FE and weights	0.000	0.019	0.024
(2) Total Expenses	Raw	-0.021	0.020	0.032
	Net of FE	0.000	0.019	0.029
	Net of FE and weights	0.000	0.010	0.007

Notes: This table presents summary statistics of the mean and dispersion of the shift-share import shock used for the empirical and quantitative analysis. Row 1 uses the definition of the shock where the denominator of the share is total imports as described in equation (41). Row 2 uses the definition of the shock where the denominator of the share is total expenses as described in equation (42). For each of the two shocks, the table displays the standard deviation and IQR for i) the raw shock, ii) the shock net of sectoral fixed effects, iii) the shock net of sectoral fixed effects, and controls for incomplete shares, as measured by the sum of weights used in the construction of a shock.

et al. (2024a) in constructing placebo shift-share shocks and run the same shock-level regressions as before with the placebo shocks instead. Specifically, we keep exposure shares fixed and draw placebo shifts. Contrary to Demir et al. (2024a), we draw shifts from the empirical distribution

Table D.5: Impact of Import Tariff Shocks on International and Domestic Production Links: Placebo

	Total Imports (1)	Number Int. Suppliers (2)	Number Dom. Suppliers (3)	Number Dom. Buyers (4)
Import Tariff Shock	-0.93 (1.63)	-1.27 (0.77)	0.53 (0.97)	0.12 (1.51)
Number Observations	33260			
Sector FE (6 digit)	Yes			
Prior Import Share	Yes			
Period	2003-2007			

Notes: This table reports the estimates of the regression equation (40) using a placebo shift-share shock where the tariff changes are drawn randomly from the empirical distribution of tariff changes. Import shocks are defined by equation (41) for Columns (1)-(2) and by equation (42) for Columns (3)-(4). All outcome variables are log changes between 2003 (pre-agreements) and 2007 (post-agreements). The samples include all Chilean firms that exist in both 2003 and 2007. Standard errors are computed following [Borusyak et al. \(2022\)](#).

of tariff changes of Chilean imports in our dataset rather than imposing any parametric assumption on the distribution of placebo shifts. We impose no structure in the randomization, except that country-products with zero changes remain unaltered. Table D.5 shows that the results are insignificant when implementing the placebo shift-share shock.

To compute standard errors when using the shift-share shocks, we proceed as follows. As in [Adão, Kolesár and Morales \(2019\)](#), BHJ shows that inference with shift-share shocks is complicated by the fact that observations with similar exposure shares probably have correlated unobservables. That is, two firms with similar exposure to changes in a product’s tariffs are more likely to have correlated unobservable shocks. To address this issue, BHJ propose a shares-weighted second stage IV regression at the shock-level, i.e., at the country-product level:

$$y_{cp} = \alpha + \beta x_{cp} + \epsilon_{cp},$$

where y_{cp} and x_{cp} are the same variables as the regression at the firm level, but appropriately weighted so that they are measured at the country-product level. In this regression, tariff changes serve as instruments for the shift-share shock x_{cp} . This approach yields correct, exposure-robust standard errors and numerically equivalent point estimates of the SSIV coefficient in a firm-level regression.

To assess the appropriate level of clustering of the shock-level regressions from above, we perform the following test. Shocks must be mutually uncorrelated, or at least across clusters. In the same spirit as in BHJ, we compute intra-class correlation coefficients of the shocks within different country-product groups. We obtain the intra-class correlations (ICC) from a random effects model hierarchically decomposing residual variation in the shifts:

$$\Delta\tau_{cp} = a_{region-1digit(cp)} + b_{region-2dig(cp)} + c_{region-3digit(cp)} + e_{cp},$$

where $a_{region-1digit(cp)}$ are country region-1 digit product code random effects, $b_{region-2digit(cp)}$ are

country region-2 digit product code random effects, and $c_{region-3digit(cp)}$ are country region-3 digit product code random effects. Table D.6 presents the estimated ICCs, suggesting that clustering is present only at the finest level of aggregation - region by 3-digit product groups. Furthermore, we cluster our standard errors for the shock-level regressions at this level, as there is little correlation across shocks at higher aggregation levels.

Table D.6: Shift-Share Import Shock: Clustering

	(1) Log Tariff Change
Country region by 1 dig product group	0.285 (0.248)
Country region by 2 dig product group	0.0645 (0.0384)
Country region by 3 dig product group	0.123 (0.0509)
Number of country-products	33260

Notes: This table presents summary statistics of the tariff changes. Estimates come from a maximum likelihood procedure with an exchangeable covariance structure for each country region-product group. Standard errors in parentheses.

E Appendix for Calibration

E.1 Adjustment of Trade Flows

As mentioned in Section 7.1, we calibrate aggregate trade flows $\{X_{ud,hk}\}$ using various data sources. However, the observed trade flows constructed in this way do not necessarily satisfy our model's equilibrium conditions. To enable well-defined counterfactuals, we adjust the trade flows consistent with the equilibrium conditions under the calibrated structural parameters $\{\alpha_k, \beta_{k,L}, \beta_{hk}, \gamma_k^B, \gamma_k^S, \varepsilon_{kl}, \lambda_{kl}^B, \lambda_{kl}^S, \mu_k, \sigma_k\}$ below by interpreting that the observed trade flows involve measurement errors.

Denote that the observed aggregate expenditure in location d and sector l by $\check{E}_{d,l} = \sum_{u,h} \check{X}_{ud,hl}$, where $\check{X}_{ud,hl}$ is observed trade flows from (u, h) to (d, l) . We assume that the actual trade flow from (u, h) to (d, l) is given by

$$X_{ud,hl} = \check{\Lambda}_{ud,hl} \beta_{hl} \check{E}_{d,l} \epsilon_{d,l}, \quad (\text{E.1})$$

where $\check{\Lambda}_{ud,hl} \equiv \frac{\check{X}_{ud,hl}}{\sum_{u'} \check{X}_{u'd,hl}}$, and $\{\epsilon_{d,l}\}$ is the measurement errors of the aggregate expenditure. We recover $\{\epsilon_{d,l}\}$ that satisfy all the market-clearing conditions. Specifically, we iterative over $\{\epsilon_{d,l}\}$ starting from initial guess until convergence: Obtain $\{X_{ud,hl}\}$ from equation (E.1)

1. Compute $\{R_{i,k}, w_i L_i, \Pi_i\}$ from equations (C.27), (C.28), (C.30), and (C.31),
2. Recompute $\{E_{i,k}\}$ using $\{R_{i,k}, w_i L_i, \Pi_i\}$ and equation (C.32)
3. Update $\{\epsilon_{i,k}\}$ by

$$\epsilon_{i,k} = \frac{E_{i,k}}{\check{E}_{i,k}},$$

where $E_{i,k}$ follows from the value in Step 2 above, $\check{E}_{i,k} = \sum_{u,h} \check{\Lambda}_{ui,hk} \beta_{hk} \check{E}_{i,k}$, and we normalize one element of $\{\epsilon_{i,k}\}$ to one.

Notice that this procedure requires the knowledge of the structural parameters $\{\alpha_k, \beta_{k,L}, \beta_{hk}, \gamma_k^B, \gamma_k^S, \varepsilon_{kl}, \lambda_{kl}^B, \lambda_{kl}^S, \mu_k, \sigma_k\}$. Therefore, when we estimate parameters $\{\lambda_{kl}^B, \lambda_{kl}^S\}$ for indirect inference (Appendix E.2), we repeat this procedure for each candidate value of $\{\lambda_{kl}^B, \lambda_{kl}^S\}$.

E.2 Estimation of λ

In this appendix, we describe the detailed estimation procedure for the matching function elasticities, $\{\lambda_{kl}^B, \lambda_{kl}^S\}$, through the indirect inference approach in Section 7.1. Recall that, due to the limited variations in tariff changes outside “goods” sectors, we assume that these parameters are common across all sectors $k, l \in K$. We also assume that the matching function elasticities are symmetric, $\lambda^B = \lambda^S \equiv \lambda$, as these two parameters tend to jointly affect the equilibrium system, making it difficult to identify each of them separately.⁷ The procedure of estimating λ is as follows.

⁷Alternative values for λ^B, λ^S while keeping the sum $\tilde{\lambda}^S + \tilde{\lambda}^B$ unchanged yields similar implications for the aggregate welfare changes (Appendix Table F.2).

1. For each sector k and origin country u , we construct the average import tariff changes $\tilde{\tau}_{uk}$ as the weighted average across HS-6 products within each category.
2. For each candidate value of λ , we undertake the counterfactual simulations of the changes in the iceberg trade costs $\hat{\tau}_{ud,kl}$ for all municipalities d and sector l within Chile. That is, we set

$$\hat{\tau}_{ud,kl} = \begin{cases} 1 & d \notin Chile \\ \Delta \ln(1 + \tilde{\tau}_{uk}) & d \in Chile \end{cases} \quad (E.2)$$

3. We run the following regression using the model-predicted outcome variables, analogous to equation (40) using actual data, but at the location and sector within Chile as a sample:

$$\ln \hat{y}_{d,l} = \beta \text{ImportTariffShock}_{d,l} + \epsilon_{d,l}. \quad (E.3)$$

Similarly, for the regression (40) using actual data, we define $\text{ImportTariffShock}_{d,l}$ as the weighted averages of $\hat{\tau}_{ud,kl}$ using baseline import shares. The outcome variable of the international import linkages is defined by

$$\text{ImportTariffShock}_{d,l} \equiv \sum_n \sum_k \frac{\text{Import}_{nd,kl}}{\text{ImportSum}_{d,l}} \times \ln \hat{\tau}_{ud,kl}. \quad (E.4)$$

The outcome variable of the domestic production linkages is defined by

$$\text{ImportTariffShock}_{d,l} \equiv \sum_n \sum_k \frac{\text{Import}_{nd,kl}}{\text{ImportSum}_{d,l} + \text{DomPurchase}_{d,l}} \times \ln \hat{\tau}_{ud,kl}. \quad (E.5)$$

These definitions align with those of the data regressions (Equations 41 and 42), correspondingly.

4. We search for λ that minimizes the squared distance between the regression coefficients on domestic supplier linkages in the data (Column 4 of Table 1) and in the model prediction. That is,

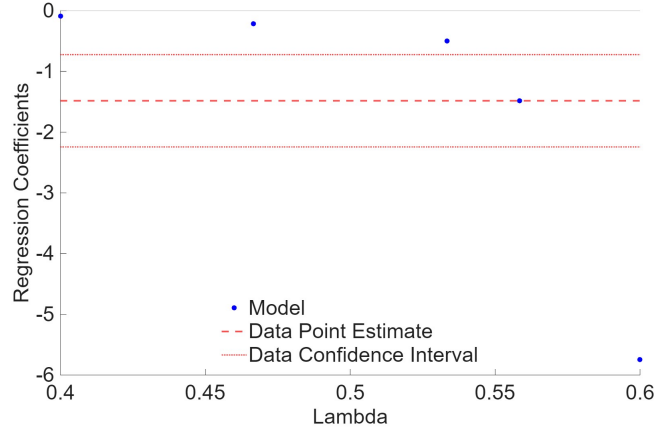
$$\hat{\lambda} \equiv \min_c \sum_c \frac{(\hat{\alpha}_c^{\text{Model}} - \hat{\alpha}_c^{\text{Data}})^2}{\text{Avar}(\hat{\alpha}_c^{\text{Data}})},$$

where c corresponds to the targeted regression coefficients Table 3, and $\text{Avar}(\hat{\alpha}_c^{\text{Data}})$ corresponds to the asymptotic variance of the regression coefficient for $\hat{\alpha}_c^{\text{Data}}$.

Weighting the regressions using the model counterfactuals using the number of firms in each municipality and sector, aligning the firm-level regression in Table 1, has virtually no effect on the regression coefficients. We cannot run the regression at the municipality and location level using our data, as it does not report the location of transactions for 2003-2007.

In Figure E.1, we show the sensitivity of λ on the targeted regression coefficient. We vary the value of λ , and show how the regression coefficients on the number of domestic suppliers in the model regression change. The targeted model coefficient is indeed responsive to these parameter values.

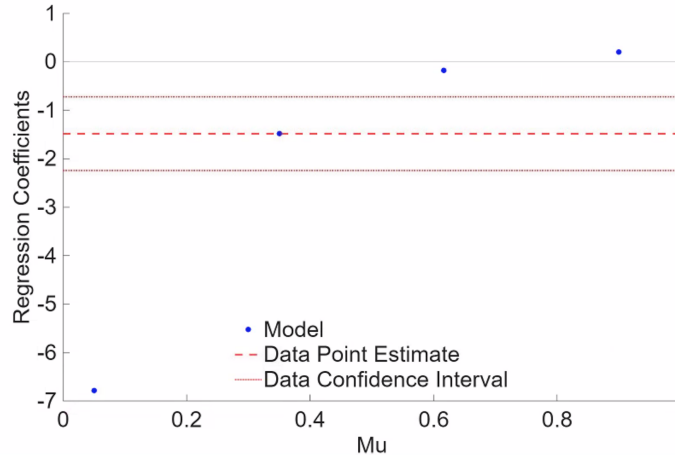
Figure E.1: Sensitivity of the impact on the number of domestic suppliers to λ



Notes: This figure presents the sensitivity of the reduced-form regression using simulations of the model of the impact of trade cost shock on the number of domestic suppliers (Y axis) to alternative values of λ (X axis). The blue dots represent the model-predicted regression coefficients using alternative values for λ . The dashed red line is the point estimate of the reduced-form coefficient using the data. The dotted red line is the confidence interval of the reduced-form coefficient using the data.

In Figure E.2, we examine how varying μ —holding all other parameters, including λ , at their calibrated baseline values—affects the regression coefficient on the number of domestic suppliers. We find that, conditional on the remaining parameters, larger values of μ make this coefficient less negative, turning positive when $\mu = 1$. This pattern aligns with the interpretation that when $\mu < 1$, higher international import costs lower the relationship-formation costs, thereby encouraging firms to add suppliers not only internationally but also domestically.

Figure E.2: Sensitivity of the impact on the number of domestic suppliers to μ



Notes: This figure presents the sensitivity of the reduced-form regression using simulations of the model of the impact of trade cost shock on the number of domestic suppliers (Y axis) to alternative values of μ (X axis). The blue dots represent the model-predicted regression coefficients using alternative values for μ . The dashed red line is the point estimate of the reduced-form coefficient using the data. The dotted red line is the confidence interval of the reduced-form coefficient using the data.

F Additional Results for Counterfactual Simulations

Table F.1: Aggregate Effects From Import Cost Increase (%): Alternative Values of σ and ε

	1) $\widehat{\text{Welfare}}$ (%)	2) Relative to Fixed Network (Row b)	3) $\hat{X}_{ui,u \in \text{US,CN}}$	4) $\hat{M}_{ui,u \in \text{US,CN}}$	5) $\hat{M}_{ui,u \in \text{CL}}$
a) Baseline	-0.88	1.6	-21.4	-8.3	-0.07
b) Fixed Networks, fix ε	-0.56	1	-21.0	0	0
c) Fixed Networks, fix σ	-0.58	1.05	-12.9	0	0

Notes: The results of counterfactual simulations to increase the iceberg trade costs from the U.S. and China to all Chilean municipalities by the same magnitude of the import tariff changes under the trade agreements used in Section 6.2, under three scenarios: (a) using our baseline parameters (Table 2), (b) shut down endogenous networks ($\lambda^S = \lambda^B = 0$), while keeping the trade elasticity ε , averaged across destination sectors, at our baseline scenario and (c) shut down endogenous networks ($\lambda^S = \lambda^B = 0$), while keeping the elasticity of substitution σ_k at our baseline scenario. Column (1) reports the changes in aggregate welfare across all Chilean municipalities (weighted average of GDP changes across Chilean municipalities with pre-shock GDP weights); Column (2) reports the ratio of the values in Column (1) to the fixed-network specification (Row b); Column (3) reports the average percent changes in imports from the U.S. and China by Chilean municipalities; Column (4) reports the average changes in the number of supplier linkages from the U.S. and China; Column (5) reports the average changes in the number of supplier linkages within Chile.

Table F.2: Aggregate Effects From Import Cost Increase (%): Alternative Values of λ

	1) $\widehat{\text{Welfare}} (\%)$	2) Relative to Fixed Network (Row b)	3) $\hat{X}_{ui,u \in \text{US,CN}}$	4) $\hat{M}_{ui,u \in \text{US,CN}}$	5) $\hat{M}_{ui,u \in \text{CL}}$
Panel 1: Main Counterfactuals					
a) Endogenous Network	-0.88	1.6	-21.4	-8.3	-0.07
b) Fixed Network, fix ε	-0.56	1	-21.0	0	0
Panel 2: Alternative Values of λ					
c) Endogenous Network, $\lambda/2$	-0.65	1.2	-21.0	-4.2	0.01
d) Endogenous Network, $\lambda^S/2$, fix $\tilde{\lambda}^S + \tilde{\lambda}^B$	-0.91	1.6	-21.3	-8.4	-0.08
e) Endogenous Network, $\lambda^B/2$, fix $\tilde{\lambda}^S + \tilde{\lambda}^B$	-0.86	1.5	-21.0	-8.0	-0.06

Notes: The results of counterfactual simulations to increase the iceberg trade costs from the U.S. and China to all Chilean municipalities by the same magnitude of the import tariff changes under the trade agreements used in Section 6.2, under two main scenarios in Panel 1: (a) using our baseline parameters (Table 2) and (b) shut down endogenous networks ($\lambda^S = \lambda^B = 0$), while keeping the trade elasticity ε , averaged across destination sectors, at our baseline scenario. In Panel 2 we explore sensitivity analysis to the calibrated model with endogenous networks under alternative values for λ : (c) by halving λ to 0.28, (d) by allowing for endogenous networks but alternatively set $\lambda^S = 0.28$, which corresponds to half its calibrated value, while keeping $\tilde{\lambda}^S + \tilde{\lambda}^B$ fixed, and (e) by allowing for endogenous networks but alternatively set $\lambda^B = 0.28$, which corresponds to half its calibrated value, while keeping $\tilde{\lambda}^S + \tilde{\lambda}^B$ fixed. Column (1) reports the changes in aggregate welfare across all Chilean municipalities (weighted average of GDP changes across Chilean municipalities with pre-shock GDP weights); Column (2) reports the ratio of the values in Column (1) to the fixed-network specification (Row b); Column (3) reports the average percent changes in imports from the U.S. and China by Chilean municipalities; Column (4) reports the average changes in the number of supplier linkages from the U.S. and China; Column (5) reports the average changes in the number of supplier linkages within Chile.