Earnings Inequality in Production Networks

Federico Huneeus
Central Bank of Chile

Kory Kroft
U. Toronto & NBER

Kevin Lim
U. Toronto

May 2023

Motivation

- There is substantial firm heterogeneity in **earnings premia** and **labor shares** of cost/revenue/VA
  - e.g. in Chile, 75/25 percentile ratio is 2.2 and 8 for earnings premia and labor share of cost

Motivation

- There is substantial firm heterogeneity in **earnings premia** and **labor shares** of cost/revenue/VA
  - e.g. in Chile, 75/25 percentile ratio is 2.2 and 8 for earnings premia and labor share of cost
- Understanding what drives this is important:
  - heterogeneity in earnings premia matters for earnings inequality across workers (Card et al. 2018)
  - heterogeneity in labor shares matters for aggregate labor shares (Autor et al. 2020)

Motivation

- There is substantial firm heterogeneity in **earnings premia** and **labor shares** of cost/revenue/VA
  - e.g. in Chile, 75/25 percentile ratio is 2.2 and 8 for earnings premia and labor share of cost
- Understanding what drives this is important:
  - heterogeneity in earnings premia matters for earnings inequality across workers (Card et al. 2018)
  - heterogeneity in labor shares matters for aggregate labor shares (Autor et al. 2020)
- We investigate how **production networks** shape earnings premia and labor shares
  - how much of earnings premia and labor share heterogeneity is explained by network heterogeneity?
  - does it matter for workers if firms grow due to networks or other drivers?

Motivation

- There is substantial firm heterogeneity in **earnings premia** and **labor shares** of cost/revenue/VA
  - e.g. in Chile, 75/25 percentile ratio is 2.2 and 8 for earnings premia and labor share of cost
- Understanding what drives this is important:
  - heterogeneity in earnings premia matters for earnings inequality across workers (Card et al. 2018)
  - heterogeneity in labor shares matters for aggregate labor shares (Autor et al. 2020)
- We investigate how **production networks** shape earnings premia and labor shares
  - how much of earnings premia and labor share heterogeneity is explained by network heterogeneity?
  - does it matter for workers if firms grow due to networks or other drivers?
- Firm performance matters for earnings premia, labor shares (e.g. Kline et al 2019, Gouin-Bonenfant 2022)
  - but focus has been on firm performance driven by innate firm characteristics such as TFP

Motivation

- There is substantial firm heterogeneity in **earnings premia** and **labor shares** of cost/revenue/VA
  - e.g. in Chile, 75/25 percentile ratio is 2.2 and 8 for earnings premia and labor share of cost
- Understanding what drives this is important:
  - heterogeneity in earnings premia matters for earnings inequality across workers (Card et al. 2018)
  - heterogeneity in labor shares matters for aggregate labor shares (Autor et al. 2020)
- We investigate how **production networks** shape earnings premia and labor shares
  - how much of earnings premia and labor share heterogeneity is explained by network heterogeneity?
  - does it matter for workers if firms grow due to networks or other drivers?
- Firm performance matters for earnings premia, labor shares (e.g. Kline et al 2019, Gouin-Bonenfant 2022)
  - but focus has been on firm performance driven by innate firm characteristics such as TFP
- Production networks matter for firm performance (e.g. Bernard et al 2018, 2022)
  - but relevance of production networks for labor market outcomes is less well-understood

Four Key Contributions of the Paper

1. **Empirical evidence** from linked employer-employee (EE) and firm-to-firm (F2F) data from Chile
    - firms with better access to customers and suppliers have higher earnings premia, lower labor shares
    - positive shocks to customer demand and supplier cost raise worker earnings
    - demand shocks have stronger effects than cost shocks conditional on growth in firm size

Four Key Contributions of the Paper

1. **Empirical evidence** from linked employer-employee (EE) and firm-to-firm (F2F) data from Chile
   - firms with better access to customers and suppliers have higher earnings premia, lower labor shares
   - positive shocks to customer demand and supplier cost raise worker earnings
   - demand shocks have stronger effects than cost shocks conditional on growth in firm size
2. **Structural model** with firm labor market power, flexible labor shares, production network linkages
   - higher demand, lower material cost in network $\Rightarrow$ higher MRPL $\Rightarrow$ higher wages
   - higher wages, lower material cost in network $\Rightarrow$ lower labor shares
   - different drivers of firm growth have different effects on labor shares and hence on wages

Four Key Contributions of the Paper

1. **Empirical evidence** from linked employer-employee (EE) and firm-to-firm (F2F) data from Chile
   - firms with better access to customers and suppliers have higher earnings premia, lower labor shares
   - positive shocks to customer demand and supplier cost raise worker earnings
   - demand shocks have stronger effects than cost shocks conditional on growth in firm size
2. **Structural model** with firm labor market power, flexible labor shares, production network linkages
   - higher demand, lower material cost in network $\Rightarrow$ higher MRPL $\Rightarrow$ higher wages
   - higher wages, lower material cost in network $\Rightarrow$ lower labor shares
   - different drivers of firm growth have different effects on labor shares and hence on wages
3. **Structural estimation** of the model leveraging EE and F2F data
   - worker-firm decomposition of earnings $\Rightarrow$ construct price index for labor
   - buyer-seller decomposition of F2F transactions $\Rightarrow$ construct price index for materials
   - use these price indices to estimate labor-materials substitution elasticity

Four Key Contributions of the Paper

1. **Empirical evidence** from linked employer-employee (EE) and firm-to-firm (F2F) data from Chile
   - firms with better access to customers and suppliers have higher earnings premia, lower labor shares
   - positive shocks to customer demand and supplier cost raise worker earnings
   - demand shocks have stronger effects than cost shocks conditional on growth in firm size
2. **Structural model** with firm labor market power, flexible labor shares, production network linkages
   - higher demand, lower material cost in network $\Rightarrow$ higher MRPL $\Rightarrow$ higher wages
   - higher wages, lower material cost in network $\Rightarrow$ lower labor shares
   - different drivers of firm growth have different effects on labor shares and hence on wages
3. **Structural estimation** of the model leveraging EE and F2F data
   - worker-firm decomposition of earnings $\Rightarrow$ construct price index for labor
   - buyer-seller decomposition of F2F transactions $\Rightarrow$ construct price index for materials
   - use these price indices to estimate labor-materials substitution elasticity
4. **Counterfactuals** to quantify importance of network for earnings premia and labor share heterogeneity
   - network explains $\frac{1}{3}$ of var(log earnings premium), $\frac{1}{4}$ of var(labor share of VA)

Related Literature

1. **Firms, labor market power and earnings inequality**: Davis and Haltiwanger (1991); Van Reenen (1996); Abowd, Kramarz, and Margolis (1999); Card, Kline and Heining (2013); Card et al (2018); Borovickova and Shimer (2018); Song et al (2019); Kline et al (2019); Bonhomme, Lamadon and Manresa (2019); Bonhomme et al (2020); Haanwinckle (2020); Lamadon, Mogstad and Setzler (2021)
   - contribution: structural representation of earnings with networks, variable labor shares

Related Literature

1. **Firms, labor market power and earnings inequality**: Davis and Haltiwanger (1991); Van Reenen (1996); Abowd, Kramarz, and Margolis (1999); Card, Kline and Heining (2013); Card et al (2018); Borovickova and Shimer (2018); Song et al (2019); Kline et al (2019); Bonhomme, Lamadon and Manresa (2019); Bonhomme et al (2020); Haanwinckle (2020); Lamadon, Mogstad and Setzler (2021)
   - contribution: structural representation of earnings with networks, variable labor shares

2. **Firms and labor share**: Karabarbounis and Neiman (2014); Autor et al (2020); Kehrig and Vincent (2021); Hubmer and Restrepo (2022); Gouin-Bonenfant (2022)
   - contribution: allow for firm earnings premia and focus on the role of intermediates

Related Literature

1. **Firms, labor market power and earnings inequality**: Davis and Haltiwanger (1991); Van Reenen (1996); Abowd, Kramarz, and Margolis (1999); Card, Kline and Heining (2013); Card et al (2018); Borovickova and Shimer (2018); Song et al (2019); Kline et al (2019); Bonhomme, Lamadon and Manresa (2019); Bonhomme et al (2020); Haanwinckle (2020); Lamadon, Mogstad and Setzler (2021)
   - contribution: structural representation of earnings with networks, variable labor shares

2. **Firms and labor share**: Karabarbounis and Neiman (2014); Autor et al (2020); Kehrig and Vincent (2021); Hubmer and Restrepo (2022); Gouin-Bonenfant (2022)
   - contribution: allow for firm earnings premia and focus on the role of intermediates

3. **Production networks**: Oberfield (2018); Huneeus (2019); Lim (2019); Alfaro-Urena et al (2019); Dhyne et al (2020); Kikkawa et al (2020); Acemoglu and Azar (2020); Demir et al (2020); Adao et al (2020); Eaton et al (2021); Dhyne et al (2022); Bernard et al (2022)
   - contribution: add heterogeneous workers and imperfectly competitive labor markets

Related Literature

1. **Firms, labor market power and earnings inequality**: Davis and Haltiwanger (1991); Van Reenen (1996); Abowd, Kramarz, and Margolis (1999); Card, Kline and Heining (2013); Card et al (2018); Borovickova and Shimer (2018); Song et al (2019); Kline et al (2019); Bonhomme, Lamadon and Manresa (2019); Bonhomme et al (2020); Haanwinckle (2020); Lamadon, Mogstad and Setzler (2021)
   - contribution: structural representation of earnings with networks, variable labor shares

2. **Firms and labor share**: Karabarbounis and Neiman (2014); Autor et al (2020); Kehrig and Vincent (2021); Hubmer and Restrepo (2022); Gouin-Bonenfant (2022)
   - contribution: allow for firm earnings premia and focus on the role of intermediates

3. **Production networks**: Oberfield (2018); Huneeus (2019); Lim (2019); Alfaro-Urena et al (2019); Dhyne et al (2020); Kikkawa et al (2020); Acemoglu and Azar (2020); Demir et al (2020); Adao et al (2020); Eaton et al (2021); Dhyne et al (2022); Bernard et al (2022)
   - contribution: add heterogeneous workers and imperfectly competitive labor markets

4. **Production function estimation**: Olley and Pakes (1996); Levinsohn and Petrin (2003); Ackerberg et al (2015),..., Doraszelski and Jaumandreu (2018)
   - contribution: new method for measuring factor prices with heterogeneous workers and inputs

Data

1. **Firm-to-Firm VAT Transactions Data**
   - frequency: annual, 2005-2010
   - coverage: all suppliers of reporting firms, all sectors ($\sim$ 80% aggregate value-added)
   - key variables: origin and destination firm tax ID, flow transaction value

Data

1. **Firm-to-Firm VAT Transactions Data**
   - frequency: annual, 2005-2010
   - coverage: all suppliers of reporting firms, all sectors ($\sim$ 80% aggregate value-added)
   - key variables: origin and destination firm tax ID, flow transaction value
2. **Matched Employer-Employee Data**
   - frequency: annual, 2005-2018
   - coverage: universe of formal private firms and their employees
   - key variables: worker earnings, monthly employment, age, gender

Data

1. **Firm-to-Firm VAT Transactions Data**
   - frequency: annual, 2005-2010
   - coverage: all suppliers of reporting firms, all sectors ($\sim$ 80% aggregate value-added)
   - key variables: origin and destination firm tax ID, flow transaction value
2. **Matched Employer-Employee Data**
   - frequency: annual, 2005-2018
   - coverage: universe of formal private firms and their employees
   - key variables: worker earnings, monthly employment, age, gender
3. **Firm Production Data**
   - frequency: monthly, 2005-2018
   - coverage: universe of formal private firms
   - key variables: sales, materials, investment, capital, main industry, HQ location

sample sizes | descriptive statistics | earnings inequality in Chile

Measurement: Firm Earnings Premia and Network Access

- To measure firm earnings premia, decompose the wage for worker m at firm i, time t:

$$\log w_{imt} = \underbrace{x_m}_{\substack{\text{worker}\\ \text{effect}}} \times \underbrace{\theta_i}_{\substack{\text{worker-firm}\\ \text{interactions}}} + \underbrace{\log f_{it}}_{\substack{\text{firm earnings}\\ \text{premium}}} + \underbrace{\hat{x}_{mt}}_{\text{residual}}$$

Measurement: Firm Earnings Premia and Network Access

- To measure firm earnings premia, decompose the wage for worker m at firm i, time t:

$$\log w_{imt} = \underbrace{x_m}_{\text{worker effect}} \times \underbrace{\theta_i}_{\text{worker-firm interactions}} + \underbrace{\log f_{it}}_{\text{firm earnings premium}} + \underbrace{\hat{x}_{mt}}_{\text{residual}}$$

- To examine role of the production network, first decompose sales by seller j to buyer i at time t:

$$\log R_{ijt} = \underbrace{\log d_{it}}_{\text{buyer effect}} + \underbrace{\log s_{jt}}_{\text{seller effect}} + \underbrace{\log e_{ijt}}_{\text{residual}}$$

Measurement: Firm Earnings Premia and Network Access

- To measure firm earnings premia, decompose the wage for worker m at firm i, time t:

$$\log w_{imt} = \underbrace{x_m}_{\substack{\text{worker}\\\text{effect}}} \times \underbrace{\theta_i}_{\substack{\text{worker-firm}\\\text{interactions}}} + \underbrace{\log f_{it}}_{\substack{\text{firm earnings}\\\text{premium}}} + \underbrace{\hat{x}_{mt}}_{\text{residual}}$$

- To examine role of the production network, first decompose sales by seller j to buyer i at time t:

$$\log R_{ijt} = \underbrace{\log d_{it}}_{\text{buyer effect}} + \underbrace{\log s_{jt}}_{\text{seller effect}} + \underbrace{\log e_{ijt}}_{\text{residual}}$$

- Then construct measures of *downstream access* D_{it}^{net} and *upstream access* S_{it}^{net}:

$$D_{it}^{net} \equiv \sum_{j \in \Omega_{it}^{C}} d_{jt} e_{jit}, \qquad S_{it}^{net} \equiv \sum_{j \in \Omega_{it}^{S}} s_{jt} e_{ijt}$$

where Ω_{it}^{C} and Ω_{it}^{S} denote the set of firm i's customers and suppliers, respectively

earnings variance decomposition | heterogeneity

Three Motivating Facts

Fact 1 Firms with greater downstream and upstream access tend to have higher firm earnings effects.

Three Motivating Facts

Fact 1 Firms with greater downstream and upstream access tend to have higher firm earnings effects.

Fact 2 Firms with greater downstream and upstream access tend to have lower labor shares of cost.

Three Motivating Facts

Fact 1 Firms with greater downstream and upstream access tend to have higher firm earnings effects.

Fact 2 Firms with greater downstream and upstream access tend to have lower labor shares of cost.

Fact 3 Demand for customers' output and cost of suppliers' inputs matter for a firm's earnings:

- higher customer demand, lower supplier input cost lead to higher earnings;
- passthrough of shocks into earnings versus value-added is incomplete; and
- customer demand has stronger effects than supplier cost conditional on the same growth in value-added.

Overview

Workers

- heterogeneous in ability a, exogenous measure $L(a)$
- derive utility from three sources:
  - consumption goods produced by firms
  - amenities offered by employer
  - idiosyncratic preferences over employers (source of market power)
- observe ability-specific wage offers made by each firm and choose employer

Overview

Workers

- heterogeneous in ability a, exogenous measure $L(a)$
- derive utility from three sources:
  - consumption goods produced by firms
  - amenities offered by employer
  - idiosyncratic preferences over employers (source of market power)
- observe ability-specific wage offers made by each firm and choose employer

Firms

- heterogeneous in factor productivities, amenity values, network connections (exogenous)
- produce output by combining workers of different abilities with materials
- set ability-specific wages to hire workers
- source materials from suppliers in production network
- sell output to final consumers and customers in network

Labor Market

- Workers spend income on consumption goods:
  - CES utility, elasticity of substitution σ, with CPI as numeraire details

Labor Market

- Workers spend income on consumption goods:
  - CES utility, elasticity of substitution σ, with CPI as numeraire details
- Workers derive idiosyncratic utilities ξ_{it} from employment at firm i
  - iid across workers and firms, distributed Gumbel (GEV-1), unobservable by firms

Labor Market

- Workers spend income on consumption goods:
  - CES utility, elasticity of substitution σ, with CPI as numeraire details
- Workers derive idiosyncratic utilities ξ_{it} from employment at firm i
  - iid across workers and firms, distributed Gumbel (GEV-1), unobservable by firms
- Utility from employment at firm i for worker with ability a and idiosyncratic utility ξ_{it}:

$$u_{it}\left(a|\xi_{it}\right) = \underbrace{\log w_{it}\left(a\right)}_{\text{wages}} + \underbrace{\log g_i\left(a\right)}_{\text{amenities}} + \underbrace{\log \tau_t}_{\text{transfer}} + \underbrace{\frac{1}{\gamma}\xi_{it}}_{\text{utility shocks}}$$

Labor Market

- Workers spend income on consumption goods:
  - CES utility, elasticity of substitution σ, with CPI as numeraire details
- Workers derive idiosyncratic utilities ξ_{it} from employment at firm i
  - iid across workers and firms, distributed Gumbel (GEV-1), unobservable by firms
- Utility from employment at firm i for worker with ability a and idiosyncratic utility ξ_{it}:

$$u_{it}(a|\xi_{it}) = \underbrace{\log w_{it}(a)}_{\text{wages}} + \underbrace{\log g_i(a)}_{\text{amenities}} + \underbrace{\log \tau_t}_{\text{transfer}} + \underbrace{\frac{1}{\gamma}\xi_{it}}_{\text{utility shocks}}$$

- Implies upward-sloping labor supply curves:

$$L_{it}(a) = \kappa_{it}(a) w_{it}(a)^{\gamma}$$

$$\kappa_{it}(a) \equiv \underbrace{L(a)}_{\text{labor stock}} \times \underbrace{\left[\Sigma_j \left(g_j(a) w_{jt}(a)\right)^{\gamma}\right]^{-1}}_{\text{labor market competition}} \times \underbrace{g_i(a)^{\gamma}}_{\text{amenities}}$$

Labor Market

- Workers spend income on consumption goods:
  - CES utility, elasticity of substitution σ, with CPI as numeraire details
- Workers derive idiosyncratic utilities ξ_{it} from employment at firm i
  - iid across workers and firms, distributed Gumbel (GEV-1), unobservable by firms
- Utility from employment at firm i for worker with ability a and idiosyncratic utility ξ_{it}:

$$u_{it}(a|\xi_{it}) = \underbrace{\log w_{it}(a)}_{\text{wages}} + \underbrace{\log g_i(a)}_{\text{amenities}} + \underbrace{\log \tau_t}_{\text{transfer}} + \underbrace{\frac{1}{\gamma}\xi_{it}}_{\text{utility shocks}}$$

- Implies upward-sloping labor supply curves:

$$L_{it}(a) = \kappa_{it}(a) w_{it}(a)^{\gamma}$$

$$\kappa_{it}(a) \equiv \underbrace{L(a)}_{\text{labor stock}} \times \underbrace{\left[\Sigma_j \left(g_j(a) w_{jt}(a)\right)^{\gamma}\right]^{-1}}_{\text{labor market competition}} \times \underbrace{g_i(a)^{\gamma}}_{\text{amenities}}$$

- Assume firms behave atomistically and perceive constant labor supply elasticity γ

Production

- Production combines labor $L_{it}(a)$ and materials $M_{it}(a)$:

$$X_{it} = T_{it} \sum_{a} F\left[\phi_i(a)\,\omega_{it} L_{it}(a), M_{it}(a)\right]$$

  - F: CES production function with elasticity of substitution ϵ extension with capital
  - T_{it}: TFP; ω_{it}: labor productivity; $\phi_i(a)$: allows for worker-firm complementarities

Production

- Production combines labor $L_{it}(a)$ and materials $M_{it}(a)$:

$$X_{it} = T_{it} \sum_a F\left[\phi_i(a)\,\omega_{it} L_{it}(a), M_{it}(a)\right]$$

  - F: CES production function with elasticity of substitution ϵ extension with capital
  - T_{it}: TFP; ω_{it}: labor productivity; $\phi_i(a)$: allows for worker-firm complementarities

- Materials produced by combining inputs from suppliers Ω_{it}^S:

$$\sum_a M_{it}(a) \equiv M_{it} = \left[\Sigma_{j\in\Omega_{it}^S} \psi_{ijt}^{\frac{1}{\sigma}} \left(x_{ijt}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

  - $\psi_{ijt} \equiv \psi_{it}\psi_{jt}\tilde{\psi}_{ijt}$: relationship-specific productivity

Production

- Production combines labor $L_{it}(a)$ and materials $M_{it}(a)$:

$$X_{it} = T_{it} \sum_a F\left[\phi_i(a)\,\omega_{it} L_{it}(a), M_{it}(a)\right]$$

  - F: CES production function with elasticity of substitution ϵ extension with capital
  - T_{it}: TFP; ω_{it}: labor productivity; $\phi_i(a)$: allows for worker-firm complementarities

- Materials produced by combining inputs from suppliers Ω^S_{it}:

$$\sum_a M_{it}(a) \equiv M_{it} = \left[\Sigma_{j\in\Omega^S_{it}} \psi_{ijt}^{\frac{1}{\sigma}} \left(x_{ijt}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

  - $\psi_{ijt} \equiv \psi_{it}\psi_{jt}\tilde{\psi}_{ijt}$: relationship-specific productivity

- Market structure: monopolistic competition
  - assume firms behave atomstically and perceive constant demand price elasticity $-\sigma$
  - optimal for each firm i to charge a common price p_{it} and markup $\mu \equiv \frac{\sigma}{\sigma-1}$ to all customers proof

Production

- Production combines labor $L_{it}(a)$ and materials $M_{it}(a)$:

$$X_{it} = T_{it} \sum_a F\left[\phi_i(a)\,\omega_{it} L_{it}(a), M_{it}(a)\right]$$

  - F: CES production function with elasticity of substitution ϵ extension with capital
  - T_{it}: TFP; ω_{it}: labor productivity; $\phi_i(a)$: allows for worker-firm complementarities

- Materials produced by combining inputs from suppliers Ω^S_{it}:

$$\sum_a M_{it}(a) \equiv M_{it} = \left[\Sigma_{j \in \Omega^S_{it}} \psi^{\frac{1}{\sigma}}_{ijt} \left(x_{ijt}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

  - $\psi_{ijt} \equiv \psi_{it}\psi_{jt}\tilde{\psi}_{ijt}$: relationship-specific productivity

- Market structure: monopolistic competition
  - assume firms behave atomstically and perceive constant demand price elasticity $-\sigma$
  - optimal for each firm i to charge a common price p_{it} and markup $\mu \equiv \frac{\sigma}{\sigma-1}$ to all customers proof

- Main departure from standard production network models: increasing marginal costs
  - hence existing models of endogenous network formation are no longer tractable

Buyer/Seller Effects, Demand, and Material Cost

- CES technology implies that sales from seller j to buyer i takes the form:

$$R_{ijt} = \underbrace{\Delta_{it}}_{\substack{\text{buyer}\\ \text{effect}}} \times \underbrace{\Phi_{jt}}_{\substack{\text{seller}\\ \text{effect}}} \times \underbrace{\psi_{ijt}}_{\substack{\text{relationship}\\ \text{productivity}}}$$

Buyer/Seller Effects, Demand, and Material Cost

- CES technology implies that sales from seller j to buyer i takes the form:

$$R_{ijt} = \underbrace{\Delta_{it}}_{\substack{\text{buyer}\\\text{effect}}} \times \underbrace{\Phi_{jt}}_{\substack{\text{seller}\\\text{effect}}} \times \underbrace{\psi_{ijt}}_{\substack{\text{relationship}\\\text{productivity}}}$$

- Buyer and seller effects $\{\Delta_{it}, \Phi_{it}\}$ for firm i are uniquely determined by:
  - firm i's own primitives, $\chi_{it} \equiv \{T_{it}, \omega_{it}, \phi_i, g_i\}$
  - firm i's demand shifter and unit cost of materials, $\{D_{it}, Z_{it}\}$

Buyer/Seller Effects, Demand, and Material Cost

- CES technology implies that sales from seller j to buyer i takes the form:

$$R_{ijt} = \underbrace{\Delta_{it}}_{\substack{\text{buyer}\\\text{effect}}} \times \underbrace{\Phi_{jt}}_{\substack{\text{seller}\\\text{effect}}} \times \underbrace{\psi_{ijt}}_{\substack{\text{relationship}\\\text{productivity}}}$$

- Buyer and seller effects $\{\Delta_{it}, \Phi_{it}\}$ for firm i are uniquely determined by:
  - firm i's own primitives, $\chi_{it} \equiv \{T_{it}, \omega_{it}, \phi_i, g_i\}$
  - firm i's demand shifter and unit cost of materials, $\{D_{it}, Z_{it}\}$

- Demand shifter and unit cost of materials are in turn determined by "network equations":

$$D_{it} = \underbrace{E_t}_{\text{final demand}} + \sum_{j \in \Omega^C_{it}} \underbrace{\Delta_{jt}\left(\chi_{jt}, D_{jt}, Z_{jt}\right)}_{\text{buyer effect of customer } j} \psi_{jit}$$

$$Z_{it}{}^{1-\sigma} = \sum_{j \in \Omega^S_{it}} \underbrace{\Phi_{jt}\left(\chi_{jt}, D_{jt}, Z_{jt}\right)}_{\text{seller effect of supplier } j} \psi_{ijt}$$

Wages

- When firms maximize profits, optimal wages are a constant markdown of MRPLs:

$$w_{it}(a) = \underbrace{\frac{\gamma}{1+\gamma}}_{\text{markdown} \equiv \eta} \times \underbrace{\phi_i(a) W_{it}}_{\text{MRPL}}$$

Wages

- When firms maximize profits, optimal wages are a constant markdown of MRPLs:

$$w_{it}(a) = \underbrace{\frac{\gamma}{1+\gamma}}_{\text{markdown} \equiv \eta} \times \underbrace{\phi_i(a) W_{it}}_{\text{MRPL}}$$

- Firm premium W_{it} is uniquely determined by own primitives χ_{it} and network variables $\{D_{it}, Z_{it}\}$ details

$$W_{it} = W(\chi_{it}, D_{it}, Z_{it})$$

Wages

- When firms maximize profits, optimal wages are a constant markdown of MRPLs:

$$w_{it}(a) = \underbrace{\frac{\gamma}{1+\gamma}}_{\text{markdown} \equiv \eta} \times \underbrace{\phi_i(a)\, W_{it}}_{\text{MRPL}}$$

- Firm premium W_{it} is uniquely determined by own primitives χ_{it} and network variables $\{D_{it}, Z_{it}\}$ details

$$W_{it} = W(\chi_{it}, D_{it}, Z_{it})$$

- Production network linkages determine $\{D_{it}, Z_{it}\}$ and hence shape wages $w_{it}(a)$ through W_{it} CD example

Four Key Theoretical Results

Result 1 The model rationalizes the reduced-form decompositions of earnings and firm-to-firm sales.

Four Key Theoretical Results

Result 1 The model rationalizes the reduced-form decompositions of earnings and firm-to-firm sales.

Result 2 The model rationalizes Fact 1 – wages $w_{it}(a)$ and the firm earnings premium W_{it} are:

- strictly increasing in downstream demand D_{it}; and
- strictly decreasing in upstream material cost Z_{it} iff $\sigma > \epsilon$.

Four Key Theoretical Results

Result 1 The model rationalizes the reduced-form decompositions of earnings and firm-to-firm sales.

Result 2 The model rationalizes Fact 1 – wages $w_{it}(a)$ and the firm earnings premium W_{it} are:

- strictly increasing in downstream demand D_{it}; and
- strictly decreasing in upstream material cost Z_{it} iff $\sigma > \epsilon$.

Result 3 The model rationalizes Fact 2 – labor shares of cost and VA are:

- strictly decreasing in downstream demand D_{it} iff $\epsilon > 1$; and
- strictly increasing in upstream material cost Z_{it} iff $\epsilon > 1$.

Four Key Theoretical Results

Result 1 The model rationalizes the reduced-form decompositions of earnings and firm-to-firm sales.

Result 2 The model rationalizes Fact 1 – wages $w_{it}(a)$ and the firm earnings premium W_{it} are:

- strictly increasing in downstream demand D_{it}; and
- strictly decreasing in upstream material cost Z_{it} iff $\sigma > \epsilon$.

Result 3 The model rationalizes Fact 2 – labor shares of cost and VA are:

- strictly decreasing in downstream demand D_{it} iff $\epsilon > 1$; and
- strictly increasing in upstream material cost Z_{it} iff $\epsilon > 1$.

Result 4 The model rationalizes Fact 3 – following a TFP, demand, or material cost shock:

- relative passthrough into wage bill versus VA is incomplete iff $\epsilon > 1$; and
- TFP, demand shocks have stronger relative passthrough than cost shocks iff $\epsilon > 1$.

Identification of Model Parameters

1.	labor supply elasticity, γ	passthrough of firm wage bill shocks to changes in worker earnings
2.	product substitution elasticity, σ	mean profit-sales ratio
3.	worker ability, a production complementarity, $\phi_i(a)$	Bonhomme et al (2019) decomposition of earnings
4.	relationship productivity, ψ_{ijt}	Bernard et al (2022) decomposition of F2F sales
5.	labor-materials substitution elasticity, ϵ labor productivity, ω_{it}	Doraszelski-Jaumandreu (2018) prod. function estimation, using 3+4 to construct factor prices
6.	amenities, $g_i(a)$	residual variation in employment shares controlling for wages
7.	TFP, T_{it}	chosen to fit firm earnings premia in 3

Estimation Results

- Estimates of key elasticities:
  - labor supply elasticity, $\gamma = 5.5$ details DiD estimate
  - labor-materials substitution elasticity, $\epsilon = 1.5$ details
  - product substitution elasticity, $\sigma = 3.1$

Estimation Results

- Estimates of key elasticities:
  - labor supply elasticity, $\gamma = 5.5$ details DiD estimate
  - labor-materials substitution elasticity, $\epsilon = 1.5$ details
  - product substitution elasticity, $\sigma = 3.1$
- Our estimates satisfy the conditions highlighted by facts 1-3 / theoretical results 2-4:
  - $\sigma > \epsilon$, so higher material cost Z_{it} leads to lower earnings premium W_{it}
  - $\epsilon > 1$, so higher demand D_{it} and lower material cost Z_{it} lead to lower labor shares $s_{it}^{L/C}, s_{it}^{L/VA}$
  - $\epsilon > 1$, so D_{it} has stronger effect on earnings than Z_{it} conditional on same growth in firm size

Estimation Results

- Estimates of key elasticities:
  - labor supply elasticity, $\gamma = 5.5$ details DiD estimate
  - labor-materials substitution elasticity, $\epsilon = 1.5$ details
  - product substitution elasticity, $\sigma = 3.1$
- Our estimates satisfy the conditions highlighted by facts 1-3 / theoretical results 2-4:
  - $\sigma > \epsilon$, so higher material cost Z_{it} leads to lower earnings premium W_{it}
  - $\epsilon > 1$, so higher demand D_{it} and lower material cost Z_{it} lead to lower labor shares $s_{it}^{L/C}$, $s_{it}^{L/VA}$
  - $\epsilon > 1$, so D_{it} has stronger effect on earnings than Z_{it} conditional on same growth in firm size
- Other results: network matching worker-firm sorting amenities

Counterfactuals: Methodology

- We now examine the drivers of four **inequality outcomes**
  1. variance of firm earnings effects: $\text{var}\left(\log W_{it}\right)$
  2. variance of labor shares of value-added: $\text{var}\left(s_{it}^{L/VA}\right)$
  3. covariance between firm earnings effects and firm size: $\text{cov}\left(\log W_{it}, \log R_{it}\right)$
  4. covariance between labor shares of value-added and firm size: $\text{cov}\left(\log s_{it}^{L/VA}, \log R_{it}\right)$

Counterfactuals: Methodology

- We now examine the drivers of four **inequality outcomes**
  1. variance of firm earnings effects: $\mathrm{var}\left(\log W_{it}\right)$
  2. variance of labor shares of value-added: $\mathrm{var}\left(s_{it}^{L/VA}\right)$
  3. covariance between firm earnings effects and firm size: $\mathrm{cov}\left(\log W_{it}, \log R_{it}\right)$
  4. covariance between labor shares of value-added and firm size: $\mathrm{cov}\left(\log s_{it}^{L/VA}, \log R_{it}\right)$
- In the model, all variances/covariances are driven by heterogeneity in five **sources of variation**
  1. network linkages (extensive + intensive margin): $\Omega_{it}^{C}, \Omega_{it}^{S}, \tilde{\psi}_{ijt}$
  2. firm productivities: $T_{it}, \omega_{it}, \psi_{it}$
  3. production complementarities: ϕ_i
  4. amenities: g_i
  5. worker abilities: a

Counterfactuals: Methodology

- We now examine the drivers of four **inequality outcomes**
  1. variance of firm earnings effects: $\text{var}(\log W_{it})$
  2. variance of labor shares of value-added: $\text{var}\left(s_{it}^{L/VA}\right)$
  3. covariance between firm earnings effects and firm size: $\text{cov}(\log W_{it}, \log R_{it})$
  4. covariance between labor shares of value-added and firm size: $\text{cov}\left(\log s_{it}^{L/VA}, \log R_{it}\right)$
- In the model, all variances/covariances are driven by heterogeneity in five **sources of variation**
  1. network linkages (extensive + intensive margin): $\Omega_{it}^{C}, \Omega_{it}^{S}, \tilde{\psi}_{ijt}$
  2. firm productivities: $T_{it}, \omega_{it}, \psi_{it}$
  3. production complementarities: ϕ_i
  4. amenities: g_i
  5. worker abilities: a
- To quantify how each source of variation contributes to each inequality outcome:
  - simulate counterfactuals with various dimensions of heterogeneity shut down
  - using a Shapley value approach to account for interdependencies in sources of variation details

Counterfactuals: Results

- **Outcome**: variance of log firm earnings effect, $\text{var}(\log W_{it})$

	(i)	(ii)	(iii)	(iv)
supplier network	23.6	21.9	35.3	-3.8
customer network	6.6	4.2	15.8	71.9
firm productivities	40.7	76.2	44.8	30.9
production complementarity	26.7	-1.7	8.0	4.5
firm amenities	13.3	-1.1	1.7	0.2
worker abilities	-10.8	0.5	-5.5	-3.8

Each column shows the percentage of an inequality outcome accounted for by each source of variation.

Counterfactuals: Results

- **Outcome**: variance of labor shares of value-added, var $\left(s_{it}^{L/VA}\right)$

	(i)	(ii)	(iii)	(iv)
supplier network	23.6	21.9	35.3	-3.8
customer network	6.6	4.2	15.8	71.9
firm productivities	40.7	76.2	44.8	30.9
production complementarity	26.7	-1.7	8.0	4.5
firm amenities	13.3	-1.1	1.7	0.2
worker abilities	-10.8	0.5	-5.5	-3.8

Each column shows the percentage of an inequality outcome accounted for by each source of variation.

Counterfactuals: Results

- **Outcome**: covariance between firm earnings effects and firm size, $\text{cov}\left(\log W_{it}, \log R_{it}\right) > 0$

	(i)	(ii)	(iii)	(iv)
supplier network	23.6	21.9	35.3	-3.8
customer network	6.6	4.2	15.8	71.9
firm productivities	40.7	76.2	44.8	30.9
production complementarity	26.7	-1.7	8.0	4.5
firm amenities	13.3	-1.1	1.7	0.2
worker abilities	-10.8	0.5	-5.5	-3.8

Each column shows the percentage of an inequality outcome accounted for by each source of variation.

Counterfactuals: Results

- **Outcome**: covariance between labor shares of value-added and firm size, $\text{cov}\left(\log s_{it}^{L/VA}, \log R_{it}\right) < 0$

	(i)	(ii)	(iii)	(iv)
supplier network	23.6	21.9	35.3	-3.8
customer network	6.6	4.2	15.8	71.9
firm productivities	40.7	76.2	44.8	30.9
production complementarity	26.7	-1.7	8.0	4.5
firm amenities	13.3	-1.1	1.7	0.2
worker abilities	-10.8	0.5	-5.5	-3.8

Each column shows the percentage of an inequality outcome accounted for by each source of variation.

Counterfactuals: Results

- In summary, production network heterogeneity accounts for:
  - 1/3 of the variation in log firm earnings effects, var($\log W_{it}$)
  - 1/4 of the variance in labor shares of value-added, $\text{var}\left(s_{it}^{L/VA}\right)$
  - 1/2 of the covariance between firm earnings effects and firm size, $\text{cov}\left(\log W_{it}, \log R_{it}\right)$
  - 2/3 of the covariance between labor shares of value-added and firm size, $\text{cov}\left(\log s_{it}^{L/VA}, \log R_{it}\right)$

Counterfactuals: Results

- In summary, production network heterogeneity accounts for:
  - 1/3 of the variation in log firm earnings effects, var(log W_{it})
  - 1/4 of the variance in labor shares of value-added, var $\left(s_{it}^{L/VA}\right)$
  - 1/2 of the covariance between firm earnings effects and firm size, cov (log W_{it}, log R_{it})
  - 2/3 of the covariance between labor shares of value-added and firm size, cov $\left(\log s_{it}^{L/VA}, \log R_{it}\right)$
- These results are also very sensitive to assuming Cobb-Douglas production functions ($\epsilon \to 1$)
  - importance of network for earnings premia heterogeneity increases by a factor of two details
  - labor shares are constant across firms

Conclusion

- Matched employer-employee and firm-to-firm datasets:
  - allow simultaneous study of disaggergated worker and firm outcomes
  - becoming more widely available to researchers (e.g. Turkey, Costa Rica, Ecuador)
- We provide a quantitative framework + estimation methodology for studying these data
  - with heterogeneous firms/workers/network and labor market power
- Network linkages matter for earnings premia and labor shares:
  - network heterogeneity explains a large share of heterogeneity in earnings premia, labor shares
  - whether firms grow through demand versus material cost matters for how workers benefit
- Extensions using the model + data:
  - automation (with Bradley Setzler) – firms have access to imported labor-replacing "robots"
  - outsourcing (with David Price) – firms hire labor or source labor indirectly from suppliers

Sample Sizes

Panel A: Firm-to-Firm Dataset	Links		Suppliers		Buyers	
Sample	**Unique**	**Observation-Years**	**Unique**	**Observation-Years**	**Unique**	**Observation-Years**
Baseline	16,831,546	31,743,495	194,615	592,622	289,344	923,155

Panel B: Employer-Employee Dataset	Workers		Firms	
Sample	**Unique**	**Observation-Years**	**Unique**	**Observations-Years**
Baseline	6,496,849	41,954,008	487,504	2,315,927
Movers	6,183,692	40,130,960	200,592	1,378,554
Stayers: Complete Spells	953,865	8,472,302	64,670	602,622
Stayers: 10 Stayers per Firm	724,957	6,571,483	5,726	61,823

Panel C: Firm Dataset	Firms	
Sample	**Unique**	**Observations-Years**
Baseline	47,685	125,726

back

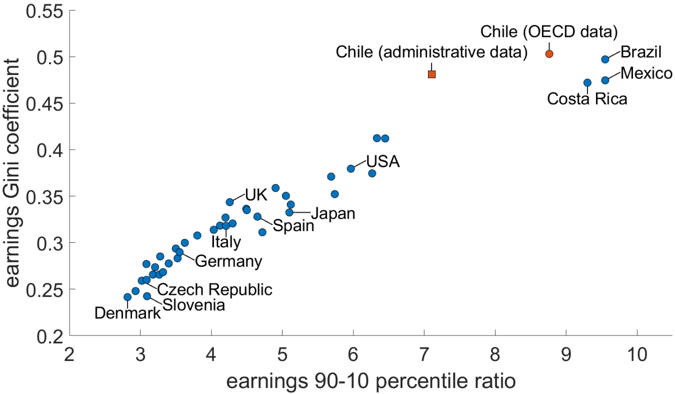
Descriptive Statistics

Dataset	Employer-Employee			Firm-to-Firm	Firm
Panel A: Worker Characteristics	**Baseline**	**Movers**	**Stayers**	**Baseline**	**Baseline**
Mean Worker Earnings (1000 US $)	11.6	11.8	17.0	10.1	9.6
Mean Worker Age	40.2	40.1	42.6	39.8	39.3
Panel B: Firm Characteristics	**Baseline**	**Movers**	**Stayers**	**Baseline**	**Baseline**
Mean Number of Workers	9	20	281	12	27
Mean Value Added (1000 US $)	59.9	133.3	2191.3	54.2	198.8
Mean Labor Share	0.49	0.45	0.70	0.49	0.42
Panel C: Production Network Characteristics	**Baseline**	**Movers**	**Stayers**	**Baseline**	**Baseline**
Mean Number of Suppliers	67	67	306	35	67
Mean Number of Buyers	80	80	580	34	80
Mean Materials Share of Sales	0.58	0.58	0.55	0.57	0.58

back

Earnings Inequality in Chile

- Earnings inequality in Chile is high by international standards:



back

Decomposition of Log Earnings Variance

- Variance of log worker earnings can be decomposed as:

$$\text{var}\left(\log w_{imt}\right) = \underbrace{\text{var}\left(\tilde{x}_m\right)}_{57\%} + \underbrace{\text{var}\left(\log \tilde{f}_{it}\right)}_{10.8\%} + \underbrace{2\text{cov}\left(\tilde{x}_m, \log \tilde{f}_{it}\right)}_{19.8\%} + \underbrace{int}_{-2.0\%} + \underbrace{\text{var}\left(\hat{x}_{mt}\right)}_{14.4\%}$$

  - $\bar{x}, \bar{\theta}$: averages of x_m, θ_i across workers
  - $\tilde{x}_m \equiv \left(x_m - \bar{x}\right)\bar{\theta}$: worker effect when employed at the average firm
  - $\log \tilde{f}_{it} \equiv \log f_{it} + \theta_i \bar{x}$: firm effect when matched with the average worker
  - int: collects terms arising from non-linear interactions between worker and firm effects

- Note that $\text{var}\left(\log \tilde{f}_{it}\right)$ is not the same as $\text{var}\left(\log f_{it}\right)$ when there are worker-firm interactions through θ_i

back

Heterogeneity in Earnings Premia, Labor Shares, and Network Access

- There is substantial heterogeneity in firm earnings premia, labor shares, and network access in Chile:

	var.	p10	p25	p50	p75	p90
log worker earnings	0.56	8.51	8.77	9.26	9.81	10.40
log firm earnings premium	0.19	0.00	0.06	0.56	0.87	1.11
labor share of cost	0.14	0.05	0.11	0.26	0.88	1.00
labor share of VA	0.32	0.09	0.17	0.33	0.61	1.28
log downstream network access	4.20	2.50	3.47	4.78	6.26	7.73
log upstream network access	3.28	2.67	3.73	4.88	5.96	7.07

- 75/25 percentile ratios:

$$\text{worker earnings: } e^{9.81-8.77} = 2.8 \qquad \text{firm earnings premium: } e^{0.87-0.06} = 2.2$$

$$\text{labor share of cost: } 0.88/0.11 = 8.0 \qquad \text{labor share of VA: } 0.61/0.17 = 3.6$$

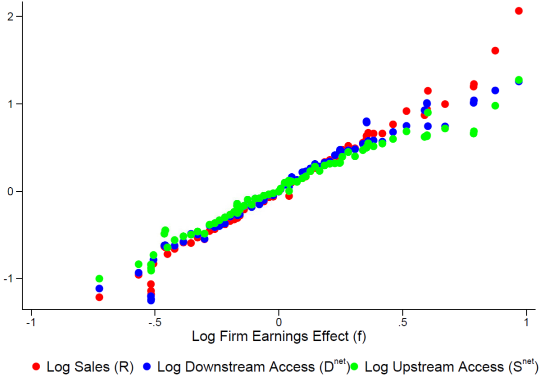
$$\text{downstream network access: } e^{6.26-3.47} = 16.3 \qquad \text{upstream network access: } e^{5.96-3.73} = 9.3$$

- $\text{cov}\,(\text{log firm earnings premium}, \text{log sales}) = 0.57$; $\text{cov}\,(\text{labor VA share}, \text{log sales}) = -0.09$

back

Fact 1: Production Network and Earnings Premia

- Firms with greater downstream and upstream access tend to have higher firm earnings effects:

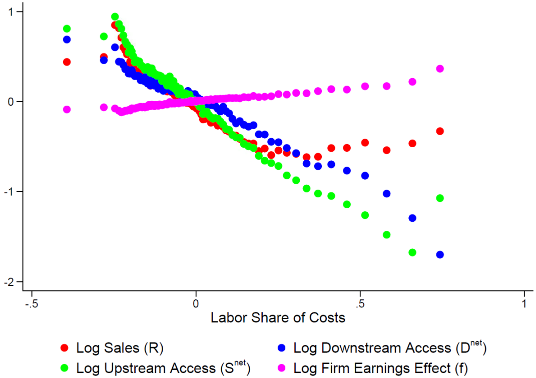


Note: All variables are parsed of industry-municipality-year means.

back

Fact 2: Production Network and Labor Shares

- Firms with greater downstream and upstream access tend to have lower labor shares of cost:



Note: All variables are parsed of industry-municipality-year means.

back

Fact 3: Passthrough of Shocks via the Network

- First define a market m as a product $\times$ foreign country pair
- Then construct export demand and import cost shocks for firm i following a shift-share design:

$$\hat{D}^{X}_{i,2010} \equiv s^{sales}_{Xi,2010} \sum_{m\in\Omega^{M,X}_{i,2005}} s^{X}_{mi,2005}\hat{s}^{I}_{m,2010}, \qquad \hat{S}^{I}_{i,2010} \equiv s^{mat}_{Ii,2010} \sum_{m\in\Omega^{M,I}_{i,2005}} s^{I}_{im,2005}\hat{s}^{X}_{m,2010}$$

  - $\hat{s}^{I}_{m,2010}, \hat{s}^{X}_{m,2010}$: log change in m's share of world imports/exports excluding Chile (2005-2010)
  - $s^{X}_{mi,2005}, s^{I}_{im,2005}$: share of i's exports/imports accounted for by m in first sample year (2005)
  - $s^{sales}_{Xi,2010}, s^{mat}_{Ii,2010}$: share of i's sales/material cost accounted for by exports/imports (2010)
  - $\Omega^{M,X}_{i,2005}, \Omega^{M,I}_{i,2005}$: markets in which i actively exports/imports in first sample year (2005)

Fact 3: Passthrough of Shocks via the Network

- Next, construct **customer** export demand shocks and **supplier** import cost shocks:

$$\hat{D}^{X,C}_{i,2010} \equiv \sum_{j \in \Omega^C_{i,2010}} s^{sales}_{ji,2010} \hat{D}^X_{j,2010}, \qquad \hat{S}^{I,S}_{i,2010} \equiv \sum_{j \in \Omega^S_{i,2010}} s^{mat}_{ij,2010} \hat{S}^I_{j,2010}$$

  - $s^{sales}_{ji,2010}$: share of seller i's sales accounted for by buyer j (2010)
  - $s^{sales}_{ji,2010}$: share of buyer i's material cost accounted for by seller j (2010)

Fact 3: Passthrough of Shocks via the Network

- Next, construct **customer** export demand shocks and **supplier** import cost shocks:

$$\hat{D}^{X,C}_{i,2010} \equiv \sum_{j \in \Omega^C_{i,2010}} s^{sales}_{ji,2010} \hat{D}^X_{j,2010}, \qquad \hat{S}^{I,S}_{i,2010} \equiv \sum_{j \in \Omega^S_{i,2010}} s^{mat}_{ij,2010} \hat{S}^I_{j,2010}$$

  - $s^{sales}_{ji,2010}$: share of seller i's sales accounted for by buyer j (2010)
  - $s^{sales}_{ji,2010}$: share of buyer i's material cost accounted for by seller j (2010)

- Then estimate the following specification via OLS:

$$\hat{Y}_{i,2010} = \alpha_D \hat{D}^{X,C}_{i,2010} + \alpha_S \hat{S}^{I,S}_{i,2010} + \beta X_i + \zeta_{i,2010}$$

  - $\hat{Y}_{it}$ = change in outcome of interest (2005-2010)
  - X_i = industry fixed effects, own export demand and import cost shocks

Fact 3: Passthrough of Shocks via the Network

	(1) Wage Bill	(2) VA
A. customer demand shocks, α_D	1.054	1.118
	(0.263)	(0.345)
B. supplier cost shocks, α_S	0.391	0.575
	(0.161)	(0.212)
industry fixed effects	yes	yes
N	27,694	27,694

- Positive effects of customer demand, supplier cost shocks on wage bill, VA
- Passthrough into wage bill vs. VA is incomplete, with higher passthrough from demand shocks:
  - demand shock: $1.054/1.118 = 94\%$
  - cost shock: $0.391/0.575 = 68\%$
- Workers do not fully capture benefits of firm growth (c.f. Berger et al 2019, Kline et al 2019)
  - and the shock driving firm growth matters
- Similar results for average wage vs. value-added per worker

back

Consumption

- Consumption utility for a worker of ability a employed at firm i:

$$\upsilon_{it}(a) = \left[\sum_{j \in \Omega^F} c_{ijt}(a)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

  - $c_{ijt}(a)$: worker's consumption of firm j's output
  - $\sigma > 1$: elasticity of substitution across products
- Take consumer price index $P_t \equiv \left(\sum_{i \in \Omega^F} p_{Fit}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ as the numeraire
  - hence consumption utility is $\upsilon_{it}(a) = \frac{w_{it}(a)\tau_t}{P_t} = w_{it}\,(a)\,\tau_t$
- Aggregate final demand for firm $i's$ output:

$$C_{it} \equiv \sum_{j \in \Omega^F} \sum_{a \in A} c_{jit}(a) L_{jt}(a) = E_t p_{Fit}^{-\sigma}$$

where $E_t = \sum_{j \in \Omega^F} \sum_{a \in A} w_{it}(a) L_{it}(a) + \sum_{i \in \Omega^F} \pi_{it}$ is aggregate consumer income

back

Capital

- Suppose firms face common capital price r and production function is:

$$X = TK^{\alpha}F\left[\{L(a), M(a)\}_{a\in A}\right]^{1-\alpha}$$

- Then define transformed demand price elasticity and TFPs:

$$\hat{\sigma} \equiv \sigma - \alpha$$

$$\hat{T} \propto \left(\frac{T}{r^{\alpha}}\right)^{\frac{\sigma-1}{\hat{\sigma}-1}}$$

- Model with capital is isomorphic to model without capital, replacing $\{\sigma, T\}$ with $\left\{\hat{\sigma}, \hat{T}\right\}$
- Note: Cobb-Douglas assumption implies that the elasticity of substitution between capital and other inputs is 1; if not, this parameter will also have to be estimated

back

Pricing

- Profit-maximization problem for firm i:

$$\max_{\{p_{jit}\}_{j\in\Omega_{it}^C\cup\{F\}}} \left\{ \sum_{j\in\Omega_{it}^C\cup\{F\}} p_{jit}x_{jit} - C\left[X_{it}|l_{it}(\cdot), Z_{it}\right] \right\}$$

$$\text{s.t. } x_{jit} = \Delta_{jt}\psi_{jit}p_{jit}^{-\sigma}$$

$$X_{it} = \sum_{j\in\Omega_{it}^C\cup\{F\}} x_{jit}$$

  - $C\left[X_{it}|l_{it}(\cdot), Z_{it}\right]$: cost of X_{it} units of output given labor supply $l_{it}(\cdot)$ and unit material cost Z_{it}
  - Δ_{jt}: demand shifter for customer j

- Optimal price charged to customer j:

$$p_{jit} = \frac{\sigma}{\sigma - 1} C'\left[X_{it}|l_{it}(\cdot), Z_{it}\right]$$

- Since marginal cost C' does not vary by customer, optimal prices do not vary by customer

back

Details: Firm Earnings Effects

- First-order conditions for labor and materials from firm profit maximization problem:

$$W_{it} = \frac{1}{\mu} D_{it}^{\frac{1}{\sigma}} X_{it}^{-\frac{1}{\sigma}} \omega_{it} T_{it} F_L(1, \nu_{it}), \qquad Z_{it} = \frac{1}{\mu} D_{it}^{\frac{1}{\sigma}} X_{it}^{-\frac{1}{\sigma}} T_{it} F_M(1, \nu_{it})$$

where $\nu_{it} \equiv \frac{M_{it}(a)}{\phi_i(a)\omega_{it}L_{it}(a)}$ is materials per efficiency unit of labor (constant across worker ability)

- Output can also be written as:

$$X_{it} = T_{it}\omega_{it}\bar{\phi}_{it}F(1, \nu_{it})(\eta W_{it})^{\gamma}$$

where $\bar{\phi}_{it} \equiv \sum_{a \in A} \kappa_{it}(a)\phi_i(a)^{1+\gamma}$ varies by firm only via amenities g_i and complementarities ϕ_i

- Given $\{D_{it}, Z_{it}, T_{it}, \omega_{it}, \bar{\phi}_{it}\}$, this defines a system of three equations in $\{W_{it}, X_{it}, \nu_{it}\}$
  - easy to show that there is a unique solution to this system

back

Example with Cobb-Douglas technology

- With Cobb-Douglas technology ($\epsilon = 1$), can rewrite firm's problem in terms of VA production function:

$$\max_{w_{it}(a)} \{ \underbrace{A_{it} \left[\sum_a \phi_i(a) L_{it}(a) \right]^{1-\alpha}}_{VA_{it} \equiv R_{it} - \sum_a Z_{it} M_{it}(a)} - \sum_a w_{it}(a) L_{it}(a) \}$$

  - curvature parameter: $\alpha \equiv \frac{1}{\sigma\lambda + (1-\lambda)}$
  - value-added productivity: $A_{it} \propto T_{it}^{\alpha(\sigma-1)} \omega_{it}^{1-\alpha} D_{it}^{\alpha} Z_{it}^{-\alpha(1-\lambda)(\sigma-1)}$
- Firm effect can be expressed as:

$$W_{it} \propto A_{it}^{\frac{1}{\alpha\gamma+1}} \tilde{\phi}_{it}^{-\frac{\alpha}{\alpha\gamma+1}}$$

  - sorting composite: $\tilde{\phi}_{it} \equiv \sum_a \kappa_{it}(a) \phi_i(a)^{1+\gamma}$
- Introduction of production network provides microfoundation for value-added productivity
  - which matters for passthrough of shocks into earnings across firms
- Value-added representation is not valid when $\epsilon \neq 1$ (we estimate $\epsilon \approx 1.5$)

back

Result 1: Structural Interpretation of Reduced-form Decompositions

- Recall the reduced-form decomposition of worker earnings:

$$\log w_{imt} = x_m \theta_i + \log f_{it} + \hat{x}_{mt}$$

Result 1: Structural Interpretation of Reduced-form Decompositions

- Recall the reduced-form decomposition of worker earnings:

$$\log w_{imt} = x_m \theta_i + \log f_{it} + \hat{x}_{mt}$$

- Suppose that ability of worker m is $a_{mt} = \{\bar{a}_m, \hat{a}_{mt}\}$
  - $\bar{a}_m$ = permanent ability; $\hat{a}_{mt}$ = iid transient ability

Result 1: Structural Interpretation of Reduced-form Decompositions

- Recall the reduced-form decomposition of worker earnings:

$$\log w_{imt} = x_m\theta_i + \log f_{it} + \hat{x}_{mt}$$

- Suppose that ability of worker m is $a_{mt} = \{\bar{a}_m, \hat{a}_{mt}\}$
  - $\bar{a}_m$ = permanent ability; $\hat{a}_{mt}$ = iid transient ability
- Suppose that productivity of worker m at firm i is:

$$\phi_i(a_{mt}) = (\bar{a}_m)^{\theta_i}\,\hat{a}_{mt}$$

Result 1: Structural Interpretation of Reduced-form Decompositions

- Recall the reduced-form decomposition of worker earnings:

$$\log w_{imt} = x_m \theta_i + \log f_{it} + \hat{x}_{mt}$$

- Suppose that ability of worker m is $a_{mt} = \{\bar{a}_m, \hat{a}_{mt}\}$
  - $\bar{a}_m$ = permanent ability; $\hat{a}_{mt}$ = iid transient ability
- Suppose that productivity of worker m at firm i is:

$$\phi_i (a_{mt}) = (\bar{a}_m)^{\theta_i} \hat{a}_{mt}$$

- Then structural interpretation of decomposition terms is:

$$f_{it} = \frac{\gamma}{1+\gamma} W_{it}, \qquad x_m = \log \bar{a}_m, \qquad \hat{x}_{mt} = \log \hat{a}_{mt}$$

Result 1: Structural Interpretation of Reduced-form Decompositions

- Recall the reduced-form decomposition of firm-to-firm sales:

$$\log R_{ijt} = \log d_{it} + \log s_{jt} + \log e_{ijt}$$

Result 1: Structural Interpretation of Reduced-form Decompositions

- Recall the reduced-form decomposition of firm-to-firm sales:

$$\log R_{ijt} = \log d_{it} + \log s_{jt} + \log e_{ijt}$$

- Suppose relationship-specific productivity is $\psi_{ijt} = \psi_{it}\psi_{jt}\tilde{\psi}_{ijt}$
  - ψ_{it} = relationship capability; $\tilde{\psi}_{ijt}$ = relationship productivity residual

Result 1: Structural Interpretation of Reduced-form Decompositions

- Recall the reduced-form decomposition of firm-to-firm sales:

$$\log R_{ijt} = \log d_{it} + \log s_{jt} + \log e_{ijt}$$

- Suppose relationship-specific productivity is $\psi_{ijt} = \psi_{it}\psi_{jt}\tilde{\psi}_{ijt}$
  - ψ_{it} = relationship capability; $\tilde{\psi}_{ijt}$ = relationship productivity residual
- Then structural interpretation of decomposition terms is:

$$d_{it} \equiv \Delta_{it}\psi_{it}, \qquad s_{jt} \equiv \Phi_{jt}\psi_{jt}, \qquad e_{ijt} \equiv \tilde{\psi}_{ijt}$$

Result 1: Structural Interpretation of Reduced-form Decompositions

- Recall the reduced-form decomposition of firm-to-firm sales:

$$\log R_{ijt} = \log d_{it} + \log s_{jt} + \log e_{ijt}$$

- Suppose relationship-specific productivity is $\psi_{ijt} = \psi_{it}\psi_{jt}\tilde{\psi}_{ijt}$
  - ψ_{it} = relationship capability; $\tilde{\psi}_{ijt}$ = relationship productivity residual
- Then structural interpretation of decomposition terms is:

$$d_{it} \equiv \Delta_{it}\psi_{it}, \qquad s_{jt} \equiv \Phi_{jt}\psi_{jt}, \qquad e_{ijt} \equiv \tilde{\psi}_{ijt}$$

- Network access measures can be recovered as:

$$D_{it}^{net} \equiv \sum_{j\in\Omega_{it}^{C}} \Delta_{jt}\psi_{jt}\tilde{\psi}_{jit}, \qquad S_{it}^{net} \equiv \sum_{j\in\Omega_{it}^{S}} \Phi_{jt}\psi_{jt}\tilde{\psi}_{ijt}$$

Result 1: Structural Interpretation of Reduced-form Decompositions

- Recall the reduced-form decomposition of firm-to-firm sales:

$$\log R_{ijt} = \log d_{it} + \log s_{jt} + \log e_{ijt}$$

- Suppose relationship-specific productivity is $\psi_{ijt} = \psi_{it}\psi_{jt}\tilde{\psi}_{ijt}$
  - ψ_{it} = relationship capability; $\tilde{\psi}_{ijt}$ = relationship productivity residual
- Then structural interpretation of decomposition terms is:

$$d_{it} \equiv \Delta_{it}\psi_{it}, \qquad s_{jt} \equiv \Phi_{jt}\psi_{jt}, \qquad e_{ijt} \equiv \tilde{\psi}_{ijt}$$

- Network access measures can be recovered as:

$$D_{it}^{net} \equiv \sum_{j \in \Omega_{it}^{C}} \Delta_{jt}\psi_{jt}\tilde{\psi}_{jit}, \qquad S_{it}^{net} \equiv \sum_{j \in \Omega_{it}^{S}} \Phi_{jt}\psi_{jt}\tilde{\psi}_{ijt}$$

- These are related to downstream demand and upstream material cost through:

$$D_{it} = E_t + \psi_{it} D_{it}^{net}, \qquad Z_{it} = \left(\psi_{it} S_{it}^{net}\right)^{\frac{1}{1-\sigma}}$$

back

Result 2: How the Network Shapes Earnings

- First-order effects of changes in demand and material cost on the earnings premium:

$$\frac{\partial \log W_{it}}{\partial \log D_{it}} = \Gamma_{it} \equiv \frac{1}{\gamma + \sigma s_{it}^{L} + \epsilon \left(1 - s_{it}^{L}\right)} > 0, \qquad \frac{\partial \log W_{it}}{\partial \log Z_{it}} = -\left(\sigma - \epsilon\right) \Gamma_{it} \left(1 - s_{it}^{L}\right)$$

Result 2: How the Network Shapes Earnings

- First-order effects of changes in demand and material cost on the earnings premium:

$$\frac{\partial \log W_{it}}{\partial \log D_{it}} = \Gamma_{it} \equiv \frac{1}{\gamma + \sigma s_{it}^{L} + \epsilon \left(1 - s_{it}^{L}\right)} > 0, \qquad \frac{\partial \log W_{it}}{\partial \log Z_{it}} = -\left(\sigma - \epsilon\right) \Gamma_{it} \left(1 - s_{it}^{L}\right)$$

- Intuition: higher demand increases the MRPL of a firm by increasing its output price p_{it}

Result 2: How the Network Shapes Earnings

- First-order effects of changes in demand and material cost on the earnings premium:

$$\frac{\partial \log W_{it}}{\partial \log D_{it}} = \Gamma_{it} \equiv \frac{1}{\gamma + \sigma s_{it}^{L} + \epsilon\left(1 - s_{it}^{L}\right)} > 0, \qquad \frac{\partial \log W_{it}}{\partial \log Z_{it}} = -\left(\sigma - \epsilon\right)\Gamma_{it}\left(1 - s_{it}^{L}\right)$$

- Intuition: higher demand increases the MRPL of a firm by increasing its output price p_{it}
- Intuition: higher material cost has both a scale and substitution effect
  - scale effect (σ): higher Z_{it} lowers the MRPL of a firm, similar to negative demand shock
  - substitution effect (ϵ): higher Z_{it} induces substitution away from materials towards labor

back

Result 3: How the Network Shapes Labor Shares

- Define labor share of cost adjusted for markdowns on wages and labor share of VA:

$$s_{it}^{L/C} \equiv \frac{\frac{1}{\eta} E_{it}^{L}}{\frac{1}{\eta} E_{it}^{L} + E_{it}^{M}}, \qquad s_{it}^{L/VA} \equiv \frac{E_{it}^{L}}{R_{it} - E_{it}^{M}}$$

  - E_{it}^{L}: labor expenditures; E_{it}^{M}: material expenditures; R_{it}: sales

Result 3: How the Network Shapes Labor Shares

- Define labor share of cost adjusted for markdowns on wages and labor share of VA:

$$s_{it}^{L/C} \equiv \frac{\frac{1}{\eta}E_{it}^{L}}{\frac{1}{\eta}E_{it}^{L} + E_{it}^{M}}, \qquad s_{it}^{L/VA} \equiv \frac{E_{it}^{L}}{R_{it} - E_{it}^{M}}$$

  - E_{it}^{L}: labor expenditures; E_{it}^{M}: material expenditures; R_{it}: sales

- Given firm profit-maximization, labor shares can be expressed as:

$$s_{it}^{L/C} = 1 - \left[1 + \left(\frac{W_{it}/\omega_{it}}{Z_{it}}\right)^{1-\epsilon}\right]^{-1}, \qquad s_{it}^{L/VA} = \frac{\eta s_{it}^{L}}{\mu - (1 - s_{it}^{L})}$$

Result 3: How the Network Shapes Labor Shares

- Define labor share of cost adjusted for markdowns on wages and labor share of VA:

$$s_{it}^{L/C} \equiv \frac{\frac{1}{\eta} E_{it}^{L}}{\frac{1}{\eta} E_{it}^{L} + E_{it}^{M}}, \qquad s_{it}^{L/VA} \equiv \frac{E_{it}^{L}}{R_{it} - E_{it}^{M}}$$

  - E_{it}^{L}: labor expenditures; E_{it}^{M}: material expenditures; R_{it}: sales

- Given firm profit-maximization, labor shares can be expressed as:

$$s_{it}^{L/C} = 1 - \left[1 + \left(\frac{W_{it}/\omega_{it}}{Z_{it}}\right)^{1-\epsilon}\right]^{-1}, \qquad s_{it}^{L/VA} = \frac{\eta s_{it}^{L}}{\mu - (1 - s_{it}^{L})}$$

- Recall that W_{it} is strictly increasing in D_{it} (Result 2)
  - can also show that W_{it}/Z_{it} is strictly decreasing in Z_{it} for any σ, ϵ

Result 3: How the Network Shapes Labor Shares

- Define labor share of cost adjusted for markdowns on wages and labor share of VA:

$$s_{it}^{L/C} \equiv \frac{\frac{1}{\eta} E_{it}^{L}}{\frac{1}{\eta} E_{it}^{L} + E_{it}^{M}}, \qquad s_{it}^{L/VA} \equiv \frac{E_{it}^{L}}{R_{it} - E_{it}^{M}}$$

  - E_{it}^{L}: labor expenditures; E_{it}^{M}: material expenditures; R_{it}: sales

- Given firm profit-maximization, labor shares can be expressed as:

$$s_{it}^{L/C} = 1 - \left[1 + \left(\frac{W_{it}/\omega_{it}}{Z_{it}}\right)^{1-\epsilon}\right]^{-1}, \qquad s_{it}^{L/VA} = \frac{\eta s_{it}^{L}}{\mu - (1 - s_{it}^{L})}$$

- Recall that W_{it} is strictly increasing in D_{it} (Result 2)
  - can also show that W_{it}/Z_{it} is strictly decreasing in Z_{it} for any σ, ϵ

- Hence $s_{it}^{L/C}, s_{it}^{L/VA}$ are strictly decreasing in D_{it} and strictly increasing in Z_{it} iff $\epsilon > 1$

back

Result 4: Firm Size, Growth, and Earnings

- The firm effect on earnings can be written in terms of sales R_{it} or value-added VA_{it} as:

$$
\begin{aligned}
\log W_{it} &= \text{const.} + \frac{1}{1+\gamma}\log R_{it} + \frac{1}{1+\gamma}\log s_{it}^{L/C} - \frac{1}{1+\gamma}\log\bar{\phi}_{it} \\
&= \text{const.} + \frac{1}{1+\gamma}\log VA_{it} + \frac{1}{1+\gamma}\log s_{it}^{L/VA} - \frac{1}{1+\gamma}\log\bar{\phi}_{it}
\end{aligned}
$$

  - $\bar{\phi}_{it}$: varies across firms only due to amenities g_i and production complementarities ϕ_i

Result 4: Firm Size, Growth, and Earnings

- The firm effect on earnings can be written in terms of sales R_{it} or value-added VA_{it} as:

$$\begin{aligned}\log W_{it} &= \text{const.} + \frac{1}{1+\gamma}\log R_{it} + \frac{1}{1+\gamma}\log s_{it}^{L/C} - \frac{1}{1+\gamma}\log \bar{\phi}_{it} \\ &= \text{const.} + \frac{1}{1+\gamma}\log VA_{it} + \frac{1}{1+\gamma}\log s_{it}^{L/VA} - \frac{1}{1+\gamma}\log \bar{\phi}_{it}\end{aligned}$$

  - $\bar{\phi}_{it}$: varies across firms only due to amenities g_i and production complementarities ϕ_i
- Firm size is not a sufficient statistic for the firm earnings effect
  - unless $\epsilon = 1$ (no variation in labor shares $s_{it}^{L/C}, s_{it}^{L/VA}$) and $\{g_i, \phi_i\}$ common across firms
- Hence decomposing firm size (Bernard et al (2022)) is not equivalent to decomposing W_{it}

Result 4: Firm Size, Growth, and Earnings

- To what extent do workers benefit when a firm grows in size?
- Consider shocks to a firm's TFP, demand shifter, or inverse unit cost of materials, $X \in \{T, D, S \equiv \frac{1}{Z}\}$

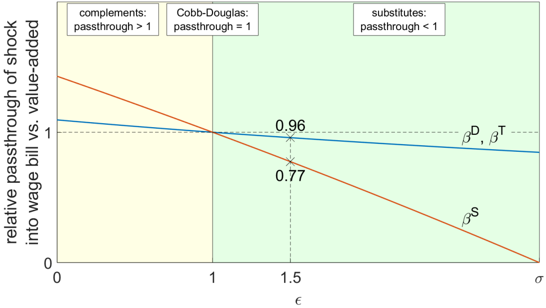
Result 4: Firm Size, Growth, and Earnings

- To what extent do workers benefit when a firm grows in size?
- Consider shocks to a firm's TFP, demand shifter, or inverse unit cost of materials, $X \in \{T, D, S \equiv \frac{1}{Z}\}$
- Relative passthrough $\beta^X \equiv \frac{\partial \log E^L_{it}}{\partial \log X_{it}} / \frac{\partial \log VA_{it}}{\partial \log X_{it}}$ into a firm's wage bill E^L_{it} versus value-added VA_{it}:
  - depends only on $s^{L/C}_{it}$, γ, σ, and ϵ details
  - for any $s^{L/C}_{it}, \gamma, \sigma$, depends critically on ϵ

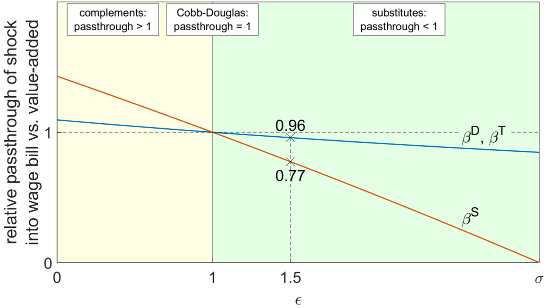
Result 4: Firm Size, Growth, and Earnings

- To what extent do workers benefit when a firm grows in size?
- Consider shocks to a firm's TFP, demand shifter, or inverse unit cost of materials, $X \in \{T, D, S \equiv \frac{1}{Z}\}$
- Relative passthrough $\beta^X \equiv \frac{\partial \log E^L_{it}}{\partial \log X_{it}} / \frac{\partial \log VA_{it}}{\partial \log X_{it}}$ into a firm's wage bill E^L_{it} versus value-added VA_{it}:
  - depends only on $s^{L/C}_{it}$, γ, σ, and ϵ details
  - for any $s^{L/C}_{it}, \gamma, \sigma$, depends critically on ϵ

Result 4: Firm Size, Growth, and Earnings

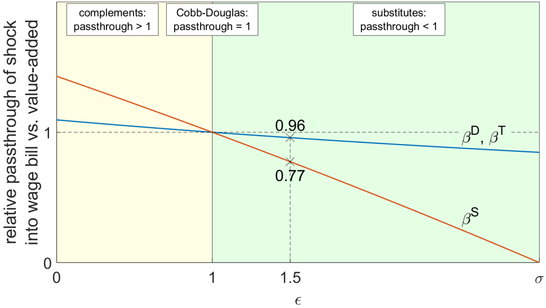


- Passthrough is incomplete if and only if $\epsilon > 1$:
  - $\{T, D, S\}$ shock $\rightarrow$ higher wages $\rightarrow$ substitution away from labor $\rightarrow$ lower wages



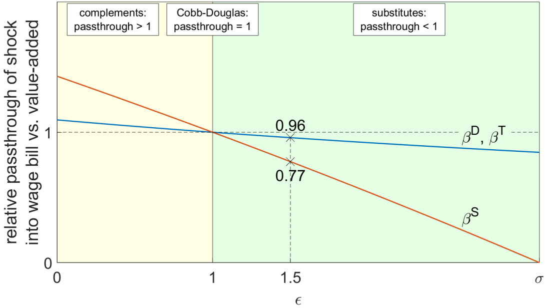
- Passthrough is incomplete if and only if $\epsilon > 1$:
  - $\{T, D, S\}$ shock $\rightarrow$ higher wages $\rightarrow$ substitution away from labor $\rightarrow$ lower wages
- When $\epsilon > 1$, D, T-driven firm growth benefits workers more than S-driven growth
  - S shocks induce a direct substitution effect away from labor toward materials

Result 4: Firm Size, Growth, and Earnings



- Passthrough is incomplete if and only if $\epsilon > 1$:
  - $\{T, D, S\}$ shock $\rightarrow$ higher wages $\rightarrow$ substitution away from labor $\rightarrow$ lower wages
- When $\epsilon > 1$, D, T-driven firm growth benefits workers more than S-driven growth
  - S shocks induce a direct substitution effect away from labor toward materials
- At estimated parameter values and median labor cost share, $\beta^D = 96\%$ (r.f. $=$ 94%), $\beta^S = 77\%$ (r.f. $=$ 68%)

Result 4: Firm Size, Growth, and Earnings



- Passthrough is incomplete if and only if $\epsilon > 1$:
  - $\{T, D, S\}$ shock $\rightarrow$ higher wages $\rightarrow$ substitution away from labor $\rightarrow$ lower wages
- When $\epsilon > 1$, D, T-driven firm growth benefits workers more than S-driven growth
  - S shocks induce a direct substitution effect away from labor toward materials
- At estimated parameter values and median labor cost share, $\beta^D = 96\%$ (r.f. $= 94\%$), $\beta^S = 77\%$ (r.f. $= 68\%$)
- Same results hold for sales instead of VA or passthrough into wages vs. value-added per worker

back

Result 4: Firm Size, Growth, and Earnings

- Passthrough into wage bill versus value-added:

$$\beta^T = \beta^D = \left[1 + \frac{(\epsilon-1)\left(1-s_{it}^{L/C}\right)}{(\gamma+1)\left[\sigma s_{it}^{L/C} + \left(1-s_{it}^{L/C}\right)\right]}\right]^{-1}, \qquad \beta^S = \left[1 + \frac{(\epsilon-1)(\gamma+\sigma)}{(\sigma-\epsilon)(\gamma+1)\left[\sigma s_{it}^{L/C} + \left(1-s_{it}^{L/C}\right)\right]}\right]^{-1}$$

- Passthrough into wage bill versus sales:

$$\beta^T = \beta^D = \left[1 + \frac{(\epsilon-1)\left(1-s_{it}^{L/C}\right)}{\gamma+1}\right]^{-1}, \qquad \beta^S = \left[1 + \frac{(\epsilon-1)(\gamma+\sigma)}{(\sigma-\epsilon)(\gamma+1)}\right]^{-1}$$

- Passthrough into earnings premium versus value-added per worker:

$$\beta^T = \beta^D = \left[1 + \frac{(\epsilon-1)\left(1-s_{it}^{L/C}\right)}{\left[\sigma s_{it}^{L/C} + \left(1-s_{it}^{L/C}\right)\right]}\right]^{-1}, \qquad \beta^S = \left[1 + \frac{(\epsilon-1)(\gamma+\sigma)}{(\sigma-\epsilon)\left[\sigma s_{it}^{L/C} + \left(1-s_{it}^{L/C}\right)\right]}\right]^{-1}$$

back

Identification: labor supply elasticity γ

- Passthrough of changes in wage bill for firm i to wages for employee m:

$$\Delta \log w_{mit} = \frac{1}{1+\gamma} \underbrace{\Delta \log E_{it}^{L}}_{\substack{\text{change in} \\ \text{firm} \\ \text{wage bill}}} + \frac{1}{1+\gamma} \underbrace{\Delta \log e_{it}^{L}}_{\substack{\text{wage bill} \\ \text{measurement} \\ \text{error}}} + \underbrace{\Delta \log \hat{a}_{mt}}_{\substack{\text{worker} \\ \text{shock}}}$$

Identification: labor supply elasticity γ

- Passthrough of changes in wage bill for firm i to wages for employee m:

$$\Delta \log w_{mit} = \frac{1}{1+\gamma} \underbrace{\Delta \log E^L_{it}}_{\substack{\text{change in} \\ \text{firm} \\ \text{wage bill}}} + \frac{1}{1+\gamma} \underbrace{\Delta \log e^L_{it}}_{\substack{\text{wage bill} \\ \text{measurement} \\ \text{error}}} + \underbrace{\Delta \log \hat{a}_{mt}}_{\substack{\text{worker} \\ \text{shock}}}$$

- Under assumption that worker and firm shocks are orthogonal, so are $\Delta \log E^L_{it}$ and $\Delta \log \hat{a}_{mt}$

Identification: labor supply elasticity γ

- Passthrough of changes in wage bill for firm i to wages for employee m:

$$\Delta \log w_{mit} = \frac{1}{1+\gamma} \underbrace{\Delta \log E^L_{it}}_{\substack{\text{change in}\\ \text{firm}\\ \text{wage bill}}} + \frac{1}{1+\gamma} \underbrace{\Delta \log e^L_{it}}_{\substack{\text{wage bill}\\ \text{measurement}\\ \text{error}}} + \underbrace{\Delta \log \hat{a}_{mt}}_{\substack{\text{worker}\\ \text{shock}}}$$

- Under assumption that worker and firm shocks are orthogonal, so are $\Delta \log E^L_{it}$ and $\Delta \log \hat{a}_{mt}$
- Suppose that:
  - measurement error in wage bills follows MA(k) process
  - time-varying firm primitives $(T_{it}, \omega_{it}, \psi_{it})$ are first-order Markov

Identification: labor supply elasticity γ

- Passthrough of changes in wage bill for firm i to wages for employee m:

$$\Delta \log w_{mit} = \frac{1}{1+\gamma} \underbrace{\Delta \log E^L_{it}}_{\substack{\text{change in} \\ \text{firm} \\ \text{wage bill}}} + \frac{1}{1+\gamma} \underbrace{\Delta \log e^L_{it}}_{\substack{\text{wage bill} \\ \text{measurement} \\ \text{error}}} + \underbrace{\Delta \log \hat{a}_{mt}}_{\substack{\text{worker} \\ \text{shock}}}$$

- Under assumption that worker and firm shocks are orthogonal, so are $\Delta \log E^L_{it}$ and $\Delta \log \hat{a}_{mt}$
- Suppose that:
  - measurement error in wage bills follows MA(k) process
  - time-varying firm primitives $(T_{it}, \omega_{it}, \psi_{it})$ are first-order Markov
- Then k and greater lags of $\Delta \log E^L_{it}$ are valid instruments

Identification: labor supply elasticity γ

- Passthrough of changes in wage bill for firm i to wages for employee m:

$$\Delta \log w_{mit} = \frac{1}{1+\gamma} \underbrace{\Delta \log E_{it}^{L}}_{\substack{\text{change in} \\ \text{firm} \\ \text{wage bill}}} + \frac{1}{1+\gamma} \underbrace{\Delta \log e_{it}^{L}}_{\substack{\text{wage bill} \\ \text{measurement} \\ \text{error}}} + \underbrace{\Delta \log \hat{a}_{mt}}_{\substack{\text{worker} \\ \text{shock}}}$$

- Under assumption that worker and firm shocks are orthogonal, so are $\Delta \log E_{it}^{L}$ and $\Delta \log \hat{a}_{mt}$
- Suppose that:
  - measurement error in wage bills follows MA(k) process
  - time-varying firm primitives $(T_{it}, \omega_{it}, \psi_{it})$ are first-order Markov
- Then k and greater lags of $\Delta \log E_{it}^{L}$ are valid instruments
- Results are robust to omitting worker m earnings from firm wage bill

Identification: labor supply elasticity γ

- Passthrough of changes in wage bill for firm i to wages for employee m:

$$\Delta \log w_{mit} = \frac{1}{1+\gamma} \underbrace{\Delta \log E^L_{it}}_{\substack{\text{change in}\\ \text{firm}\\ \text{wage bill}}} + \frac{1}{1+\gamma} \underbrace{\Delta \log e^L_{it}}_{\substack{\text{wage bill}\\ \text{measurement}\\ \text{error}}} + \underbrace{\Delta \log \hat{a}_{mt}}_{\substack{\text{worker}\\ \text{shock}}}$$

- Under assumption that worker and firm shocks are orthogonal, so are $\Delta \log E^L_{it}$ and $\Delta \log \hat{a}_{mt}$
- Suppose that:
  - measurement error in wage bills follows MA(k) process
  - time-varying firm primitives $(T_{it}, \omega_{it}, \psi_{it})$ are first-order Markov
- Then k and greater lags of $\Delta \log E^L_{it}$ are valid instruments
- Results are robust to omitting worker m earnings from firm wage bill
- Note that using value-added shocks instead of wage bill shocks is not valid:
  - unless there is no output market power (profits) or no materials, so $E^L_{it} \propto VA_{it}$

back

Identification: elasticity of substitution σ

- The first-order conditions from the firm's profit maximization problem imply:

$$\underbrace{R_{it}}_{\text{sales}} = \underbrace{\frac{\sigma}{\sigma - 1}}_{\text{markup}} \times \underbrace{\left[\frac{1}{\eta} E_{it}^{L} + E_{it}^{M}\right]}_{\text{production cost adjusted for wage markdown}}$$

where η corrects for wage markdown and increasing marginal cost

Identification: elasticity of substitution σ

- The first-order conditions from the firm's profit maximization problem imply:

$$\underbrace{R_{it}}_{\text{sales}} = \underbrace{\frac{\sigma}{\sigma - 1}}_{\text{markup}} \times \underbrace{\left[\frac{1}{\eta} E^L_{it} + E^M_{it}\right]}_{\text{production cost adjusted for wage markdown}}$$

where η corrects for wage markdown and increasing marginal cost

- Hence we identify σ from the following moment condition:

$$\sigma = \mathbb{E}\left[\frac{R_{it}}{R_{it} - \frac{1}{\eta} E^L_{it} - E^M_{it}}\right]$$

interpreting empirical deviations from the FOC as measurement error

Identification: elasticity of substitution σ

- The first-order conditions from the firm's profit maximization problem imply:

$$\underbrace{R_{it}}_{\text{sales}} = \underbrace{\frac{\sigma}{\sigma - 1}}_{\text{markup}} \times \underbrace{\left[\frac{1}{\eta} E^L_{it} + E^M_{it}\right]}_{\text{production cost adjusted for wage markdown}}$$

  where η corrects for wage markdown and increasing marginal cost
- Hence we identify σ from the following moment condition:

$$\sigma = \mathbb{E}\left[\frac{R_{it}}{R_{it} - \frac{1}{\eta} E^L_{it} - E^M_{it}}\right]$$

  interpreting empirical deviations from the FOC as measurement error
- Intuition: if firms make high profit fixing output, then demand must be inelastic
  - when $\gamma \to \infty$, $\eta \to 1$ and σ is identified from the population average sales-profit ratio

back

Identification: worker and firm effects

- First assume form for production complementarities: $\phi_i(a) = \bar{a}^{\theta_i} \times \hat{a}$

Identification: worker and firm effects

- First assume form for production complementarities: $\phi_i(a) = \bar{a}^{\theta_i} \times \hat{a}$
- Wage equation is then:

$$\log \tilde{w}_{imt} = \underbrace{\theta_i \log \bar{a}_m}_{\text{worker-firm interaction}} + \underbrace{\log W_i}_{\text{firm FE}} + \underbrace{\log \hat{a}_{mt}}_{\text{residual}}$$

where $\log \tilde{w}_{imt} = \log w_{imt} - \frac{1}{1+\gamma}\left(\log E^L_{it} - \mathbb{E}_t\left[\log E^L_{it}\right]\right)$ and $\log W_i \equiv \mathbb{E}_t\left[\log \eta W_{it}\right]$

Identification: worker and firm effects

- First assume form for production complementarities: $\phi_i(a) = \bar{a}^{\theta_i} \times \hat{a}$
- Wage equation is then:

$$\log \tilde{w}_{imt} = \underbrace{\theta_i \log \bar{a}_m}_{\text{worker-firm interaction}} + \underbrace{\log W_i}_{\text{firm FE}} + \underbrace{\log \hat{a}_{mt}}_{\text{residual}}$$

where $\log \tilde{w}_{imt} = \log w_{imt} - \frac{1}{1+\gamma}\left(\log E_{it}^{L} - \mathbb{E}_t\left[\log E_{it}^{L}\right]\right)$ and $\log W_i \equiv \mathbb{E}_t\left[\log \eta W_{it}\right]$

- Bonhomme et al (2019) show that $\{\theta_i, W_i\}$ are identified from:

$$E\left[\frac{1}{\theta_j}\left(\log \tilde{w}_{jm,t+1} - \log W_j\right) - \frac{1}{\theta_i}\left(\log \tilde{w}_{im,t} - \log W_i\right) | m \in M_{t,t+1}^{i\to j}\right] = 0$$

where $M_{t,t+1}^{i\to j}$ is the set of workers that move from i to j between t and $t+1$

Identification: worker and firm effects

- First assume form for production complementarities: $\phi_i(a) = \bar{a}^{\theta_i} \times \hat{a}$
- Wage equation is then:

$$\log \tilde{w}_{imt} = \underbrace{\theta_i \log \bar{a}_m}_{\text{worker-firm interaction}} + \underbrace{\log W_i}_{\text{firm FE}} + \underbrace{\log \hat{a}_{mt}}_{\text{residual}}$$

  where $\log \tilde{w}_{imt} = \log w_{imt} - \frac{1}{1+\gamma}\left(\log E^L_{it} - \mathbb{E}_t\left[\log E^L_{it}\right]\right)$ and $\log W_i \equiv \mathbb{E}_t\left[\log \eta W_{it}\right]$

- Bonhomme et al (2019) show that $\{\theta_i, W_i\}$ are identified from:

$$E\left[\frac{1}{\theta_j}\left(\log \tilde{w}_{jm,t+1} - \log W_j\right) - \frac{1}{\theta_i}\left(\log \tilde{w}_{im,t} - \log W_i\right) | m \in M^{i\to j}_{t,t+1}\right] = 0$$

  where $M^{i\to j}_{t,t+1}$ is the set of workers that move from i to j between t and $t+1$

- Follow BLM by restricting $\{\theta_i, W_i\}$ to vary only by K clusters so that $\theta_i = \theta_{k(i)}$ and $W_i = W_{k(i)}$

Identification: worker and firm effects

- First assume form for production complementarities: $\phi_i(a) = \bar{a}^{\theta_i} \times \hat{a}$
- Wage equation is then:

$$\log \tilde{w}_{imt} = \underbrace{\theta_i \log \bar{a}_m}_{\text{worker-firm interaction}} + \underbrace{\log W_i}_{\text{firm FE}} + \underbrace{\log \hat{a}_{mt}}_{\text{residual}}$$

  where $\log \tilde{w}_{imt} = \log w_{imt} - \frac{1}{1+\gamma}\left(\log E^L_{it} - \mathbb{E}_t\left[\log E^L_{it}\right]\right)$ and $\log W_i \equiv \mathbb{E}_t\left[\log \eta W_{it}\right]$

- Bonhomme et al (2019) show that $\{\theta_i, W_i\}$ are identified from:

$$E\left[\frac{1}{\theta_j}\left(\log \tilde{w}_{jm,t+1} - \log W_j\right) - \frac{1}{\theta_i}\left(\log \tilde{w}_{im,t} - \log W_i\right) | m \in M^{i\to j}_{t,t+1}\right] = 0$$

  where $M^{i\to j}_{t,t+1}$ is the set of workers that move from i to j between t and $t+1$

- Follow BLM by restricting $\{\theta_i, W_i\}$ to vary only by K clusters so that $\theta_i = \theta_{k(i)}$ and $W_i = W_{k(i)}$
- Given identification of $\left\{\theta_{k(i)}, W_{k(i)}\right\}$, identify permanent worker ability as $\log \bar{a}_m = E\left[\frac{\log \tilde{w}_{imt} - \log W_{k(i)}}{\theta_{k(i)}}\right]$

Identification: worker and firm effects

- First assume form for production complementarities: $\phi_i(a) = \bar{a}^{\theta_i} \times \hat{a}$
- Wage equation is then:

$$\log \tilde{w}_{imt} = \underbrace{\theta_i \log \bar{a}_m}_{\text{worker-firm interaction}} + \underbrace{\log W_i}_{\text{firm FE}} + \underbrace{\log \hat{a}_{mt}}_{\text{residual}}$$

where $\log \tilde{w}_{imt} = \log w_{imt} - \frac{1}{1+\gamma}\left(\log E^L_{it} - \mathbb{E}_t\left[\log E^L_{it}\right]\right)$ and $\log W_i \equiv \mathbb{E}_t\left[\log \eta W_{it}\right]$

- Bonhomme et al (2019) show that $\{\theta_i, W_i\}$ are identified from:

$$E\left[\frac{1}{\theta_j}\left(\log \tilde{w}_{jm,t+1} - \log W_j\right) - \frac{1}{\theta_i}\left(\log \tilde{w}_{im,t} - \log W_i\right) | m \in M^{i \to j}_{t,t+1}\right] = 0$$

where $M^{i \to j}_{t,t+1}$ is the set of workers that move from i to j between t and $t+1$

- Follow BLM by restricting $\{\theta_i, W_i\}$ to vary only by K clusters so that $\theta_i = \theta_{k(i)}$ and $W_i = W_{k(i)}$
- Given identification of $\{\theta_{k(i)}, W_{k(i)}\}$, identify permanent worker ability as $\log \bar{a}_m = E\left[\frac{\log \tilde{w}_{imt} - \log W_{k(i)}}{\theta_{k(i)}}\right]$
- Time-varying firm effect W_{it} recovered using $\log W_{it} = \log W_{k(i)} + \frac{1}{1+\gamma}\left(\log E^L_{it} - \mathbb{E}_t\left[\log E^L_{it}\right]\right)$

back

Identification: relationship capability

- First assume decomposition of relationship productivity: $\psi_{ijt} = \psi_{it}\psi_{jt}\tilde{\psi}_{ijt}$

Identification: relationship capability

- First assume decomposition of relationship productivity: $\psi_{ijt} = \psi_{it}\psi_{jt}\tilde{\psi}_{ijt}$
- Then sales from firm j to firm i are:

$$\log R_{ijt} = \underbrace{\log \tilde{\Delta}_{it}}_{\text{buyer effect}} + \underbrace{\log \tilde{\Phi}_{jt}}_{\text{seller effect}} + \underbrace{\log \tilde{\psi}_{ijt}}_{\text{residual}}$$

where $\tilde{\Delta}_{it} \equiv \Delta_{it}\psi_{it}$ and $\tilde{\Phi}_{jt} \equiv \Phi_{jt}\psi_{jt}$ are (transformed) buyer and seller effects

Identification: relationship capability

- First assume decomposition of relationship productivity: $\psi_{ijt} = \psi_{it}\psi_{jt}\tilde{\psi}_{ijt}$
- Then sales from firm j to firm i are:

$$\log R_{ijt} = \underbrace{\log \tilde{\Delta}_{it}}_{\text{buyer effect}} + \underbrace{\log \tilde{\Phi}_{jt}}_{\text{seller effect}} + \underbrace{\log \tilde{\psi}_{ijt}}_{\text{residual}}$$

where $\tilde{\Delta}_{it} \equiv \Delta_{it}\psi_{it}$ and $\tilde{\Phi}_{jt} \equiv \Phi_{jt}\psi_{jt}$ are (transformed) buyer and seller effects

- Following Bernard et al (2022), assume buyer-seller matching is independent of $\tilde{\psi}_{ijt}$
  - hence $\mathbb{E}\left[\log \tilde{\Delta}_{it} log \tilde{\psi}_{ijt}\right] = \mathbb{E}\left[\log \tilde{\Phi}_{jt} log \tilde{\psi}_{ijt}\right] = 0$

Identification: relationship capability

- First assume decomposition of relationship productivity: $\psi_{ijt} = \psi_{it}\psi_{jt}\tilde{\psi}_{ijt}$
- Then sales from firm j to firm i are:

$$\log R_{ijt} = \underbrace{\log \tilde{\Delta}_{it}}_{\text{buyer effect}} + \underbrace{\log \tilde{\Phi}_{jt}}_{\text{seller effect}} + \underbrace{\log \tilde{\psi}_{ijt}}_{\text{residual}}$$

  where $\tilde{\Delta}_{it} \equiv \Delta_{it}\psi_{it}$ and $\tilde{\Phi}_{jt} \equiv \Phi_{jt}\psi_{jt}$ are (transformed) buyer and seller effects
- Following Bernard et al (2022), assume buyer-seller matching is independent of $\tilde{\psi}_{ijt}$
  - hence $\mathbb{E}\left[\log \tilde{\Delta}_{it} log \tilde{\psi}_{ijt}\right] = \mathbb{E}\left[\log \tilde{\Phi}_{jt} log \tilde{\psi}_{ijt}\right] = 0$
- To recover ψ_{it}, use share of i's total sales s_{it}^{net} from network (excluding final sales):

$$\psi_{it} = E_t\left(\frac{s_{it}^{net}}{1 - s_{it}^{net}}\right) \frac{1}{\sum_{j\in\Omega_{it}^{C}} \tilde{\Delta}_{jt}\tilde{\psi}_{jit}}$$

- Note that we only need to identify ψ_{it} up to a constant since we have T_{it}, ω_{it}
- Given ψ_{it}, can recover buyer and seller effects Δ_{it}, Φ_{it}

back

Identification: labor-materials substitution elasticity ϵ

- Standard CES production function with labor-augmenting productivity ω_{it} implies:

$$\underbrace{\log\left(E_{it}^{M}/E_{it}^{L}\right)}_{\substack{\text{relative M-L}\\ \text{expenditure}}} = \text{const.} + (1-\epsilon)\underbrace{\log\left(P_{it}^{M}/P_{it}^{L}\right)}_{\substack{\text{relative M-L}\\ \text{unit price}}} + (1-\epsilon)\log\omega_{it}$$

Identification: labor-materials substitution elasticity ϵ

- Standard CES production function with labor-augmenting productivity ω_{it} implies:

$$\underbrace{\log\left(E_{it}^{M}/E_{it}^{L}\right)}_{\substack{\text{relative M-L} \\ \text{expenditure}}} = \text{const.} + (1-\epsilon)\underbrace{\log\left(P_{it}^{M}/P_{it}^{L}\right)}_{\substack{\text{relative M-L} \\ \text{unit price}}} + (1-\epsilon)\log\omega_{it}$$

- Given input prices $\{P_{it}^{M}, P_{it}^{L}\}$, Doraszelski and Jaumandreu (2018) develop approach to identify $\{\epsilon, \omega_{it}\}$
  - instrument prices with lagged prices and expenditures
  - use control function in lagged prices and expenditures to control for ω_{it}

Identification: labor-materials substitution elasticity ϵ

- Standard CES production function with labor-augmenting productivity ω_{it} implies:

$$\underbrace{\log\left(E_{it}^{M}/E_{it}^{L}\right)}_{\substack{\text{relative M-L}\\ \text{expenditure}}} = \text{const.} + (1-\epsilon)\underbrace{\log\left(P_{it}^{M}/P_{it}^{L}\right)}_{\substack{\text{relative M-L}\\ \text{unit price}}} + (1-\epsilon)\log\omega_{it}$$

- Given input prices $\{P_{it}^{M}, P_{it}^{L}\}$, Doraszelski and Jaumandreu (2018) develop approach to identify $\{\epsilon, \omega_{it}\}$
  - instrument prices with lagged prices and expenditures
  - use control function in lagged prices and expenditures to control for ω_{it}
- What are the correct price measures when **both workers and inputs are heterogeneous**?

Identification: labor-materials substitution elasticity ϵ

- Standard CES production function with labor-augmenting productivity ω_{it} implies:

$$\underbrace{\log\left(E_{it}^{M}/E_{it}^{L}\right)}_{\substack{\text{relative M-L}\\\text{expenditure}}} = \text{const.} + (1-\epsilon)\underbrace{\log\left(P_{it}^{M}/P_{it}^{L}\right)}_{\substack{\text{relative M-L}\\\text{unit price}}} + (1-\epsilon)\log\omega_{it}$$

- Given input prices $\{P_{it}^{M}, P_{it}^{L}\}$, Doraszelski and Jaumandreu (2018) develop approach to identify $\{\epsilon, \omega_{it}\}$
  - instrument prices with lagged prices and expenditures
  - use control function in lagged prices and expenditures to control for ω_{it}
- What are the correct price measures when **both workers and inputs are heterogeneous**?
- Current literature approach to measurement of input prices:
  - P_{it}^{L} = avg. local market wage (e.g. Oberfield-Raval (2020)), avg. firm wage (e.g. DJ (2018))
  - P_{it}^{M} = industry characteristic (e.g. Oberfield-Raval (2020)); self-reported price (e.g. DJ (2018))

Identification: labor-materials substitution elasticity ϵ

- "Price of labor" can be estimated from decomposition of worker earnings into worker and firm effects:

$$\log w_{mit} = \log \eta + \underbrace{\theta_i \log \bar{a}_m}_{\substack{\text{worker-firm} \\ \text{interaction}}} + \underbrace{\log W_{it}}_{\substack{\text{firm} \\ \text{premium}}} + \underbrace{\log \hat{a}_{mt}}_{\substack{\text{worker} \\ \text{residual}}}$$

  - theoretically correct price of labor is $P^L_{it} = W_{it}$, i.e. the firm premium

Identification: labor-materials substitution elasticity ϵ

- "Price of labor" can be estimated from decomposition of worker earnings into worker and firm effects:

$$\log w_{mit} = \log \eta + \underbrace{\theta_i \log \bar{a}_m}_{\substack{\text{worker-firm}\\ \text{interaction}}} + \underbrace{\log W_{it}}_{\substack{\text{firm}\\ \text{premium}}} + \underbrace{\log \hat{a}_{mt}}_{\substack{\text{worker}\\ \text{residual}}}$$

  - theoretically correct price of labor is $P^L_{it} = W_{it}$, i.e. the firm premium

- "Price of materials" can be estimated from decomposition of firm-to-firm sales into buyer and seller effects:

$$\log R_{ijt} = \underbrace{\log \Delta_{it}}_{\substack{\text{buyer}\\ \text{effect}}} + \underbrace{\log \Phi_{jt}}_{\substack{\text{seller}\\ \text{effect}}} + \underbrace{\log \psi_{ijt}}_{\substack{\text{relationship}\\ \text{productivity}}}$$

  - theoretically correct price of materials is $P^M_{it} = Z_{it} = \left[\sum_{j \in \Omega^S_{it}} \Phi_{jt} \psi_{ijt}\right]^{\frac{1}{1-\sigma}}$, i.e. aggregation of seller effects across suppliers adjusted by relationship productivity

back

Identification: amenities $g_i(a)$

- Follow LMS (2021) in restricting amenities as follows:

$$g_i(a) = \tilde{g}_i \bar{g}_{k(i)}(\bar{a})$$

Identification: amenities $g_i(a)$

- Follow LMS (2021) in restricting amenities as follows:

$$g_i(a) = \tilde{g}_i \bar{g}_{k(i)}(\bar{a})$$

- Cluster-ability component of amenities can be identified from:

$$\bar{g}_{k(i)}(\bar{a}) = (\bar{a})^{-\theta_k} \left[\Lambda_{kt}(\bar{a})\right]^{\frac{1}{\gamma}}$$

where $\Lambda_{kt}(\bar{a})$ is share of workers of permanent ability $\bar{a}$ employed by firms in cluster k

Identification: amenities $g_i(a)$

- Follow LMS (2021) in restricting amenities as follows:

$$g_i(a) = \tilde{g}_i \bar{g}_{k(i)}(\bar{a})$$

- Cluster-ability component of amenities can be identified from:

$$\bar{g}_{k(i)}(\bar{a}) = (\bar{a})^{-\theta_k} \left[\Lambda_{kt}(\bar{a})\right]^{\frac{1}{\gamma}}$$

  where $\Lambda_{kt}(\bar{a})$ is share of workers of permanent ability $\bar{a}$ employed by firms in cluster k

- Firm-specific component of amenities can be identified from:

$$\tilde{g}_i = \frac{1}{W_{it}} \left(\frac{\bar{\Lambda}_{it}}{\bar{\Lambda}_{k(i)t}} \right)^{\frac{1}{\gamma}}$$

  where $\bar{\Lambda}_{it}$ and $\bar{\Lambda}_{k(i)t}$ are shares of employment (of all worker types) accounted for by firm i and cluster $k(i)$

back

Identification: firm TFP T_{it}

- We can express the time-varying firm effects that we recover from BLM as:

$$W_{it} = F_i \left[\{ T_{jt} \}_{j \in \Omega^F} | \Theta_{-T} \right]$$

  - Θ_{-T}: set of model primitives other than TFPs
  - $\{F_i\}_{i \in \Omega^F}$: set of known functions that depend on structural relationships of model

Identification: firm TFP T_{it}

- We can express the time-varying firm effects that we recover from BLM as:

$$W_{it} = F_i \left[\{ T_{jt} \}_{j \in \Omega^F} | \Theta_{-T} \right]$$

  - Θ_{-T}: set of model primitives other than TFPs
  - $\{F_i\}_{i \in \Omega^F}$: set of known functions that depend on structural relationships of model
- Given identification of Θ_{-T}, this provides a set of $\left|\Omega^F\right|$ moments for exact identification of TFP
- Note that without intermediates ($\lambda \to 1$), log W_{it} is linear in log T_{it} and identification is straightforward
- With intermediates, F_i is generally not log-linear and depends on T_{jt} for $j \neq i$
  - hence need numerical approach in practice for estimation

back

Estimation of Labor Supply Elasticity

	$\Delta \log w_{imt}$		
	(1)	(2)	(3)
$\Delta \log \tilde{E}^L_{it}$	0.155 (0.006)	0.177 (0.007)	0.268 (0.001)
γ	5.5	4.6	2.7
Strategy	GMM	GMM	OLS
Instruments Accumulated Lags	5	3	
First Stage F-Stat	2325	1426	
Number of Observations	2,507,868	2,507,868	2,507,868

Notes: This table presents results from the passthrough regression used to estimate the labor supply elasticity γ. All GMM strategies use different instruments of cubic polynomials of lags of wage bill and is implemented in two stages with a robust weighting matrix used to compute standard errors. Column 1 (our preferred specification) uses changes of wage bill lagged for 3, 4 and 5 periods as instruments. Column 2 uses changes of wage bill lagged for 3 periods as instruments. Column 3 estimates the model with OLS, which ignores measurement error on the wage bill. Standard errors are shown in parentheses.

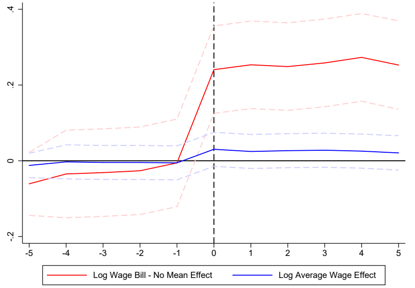
back

Difference-in-Difference Estimate of Labor Supply Elasticity

- Following Lamadon et al (2021), we also estimate γ using a difference-in-difference approach
  - for each year, order firms according to log changes in wage bill
  - treated group: firms with above-median log wage bill changes
  - plot difference in wage bill of treated and control firms
  - do this for each calendar year and weight firms by the number of workers

Difference-in-Difference Estimate of Labor Supply Elasticity

- Following Lamadon et al (2021), we also estimate γ using a difference-in-difference approach
  - for each year, order firms according to log changes in wage bill
  - treated group: firms with above-median log wage bill changes
  - plot difference in wage bill of treated and control firms
  - do this for each calendar year and weight firms by the number of workers
- Implied labor supply elasticity estimate is $\hat{\gamma} = 5.5$, same as baseline



back

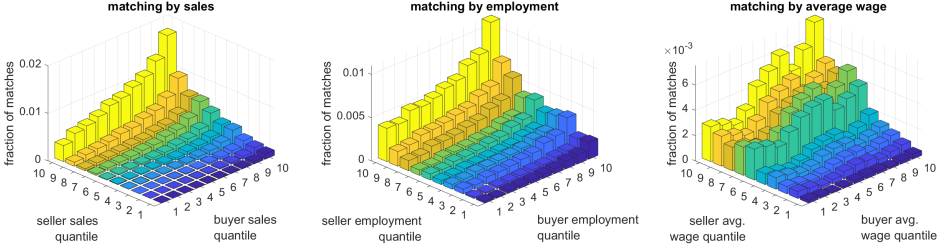
Estimation of Labor-Materials Substitution Elasticity

	$\log E^M/E^L$		
	(1)	(2)	(3)
$\log Z/W$	-0.553	-0.623	
	(0.058)	(0.094)	
$\log Z/\bar{w}$			-0.052
			(0.043)
ϵ	1.55	1.62	1.05
Model for Wage Component	BLM	AKM	Average
Instruments	$\{E^M_{it-1}, E^L_{it-1}\}$	$\{E^M_{it-1}, E^L_{it-1}, W_{it-1}, Z_{it-1}\}$	$\{W_{it-1}, Z_{it-1}\}$
Instrument Polynomial	Quadratic	Linear	Quadratic
First Stage F-Stat	130	84	186
Hansen's J Test	0.121	0.379	0.003
Number of Observations	44,967	44,967	44,967

Notes: This table presents estimates of the labor-materials subtitution elasticity ϵ. Column 1 is our preferred specification. Column 2 uses the AKM wage model to estimate the firm effect W_{it} while Column 3 uses the average firm wage instead of W_{it}. All specifications are estimated using two-stage GMM with a robust weighting matrix. Standard errors are shown in parentheses.

back

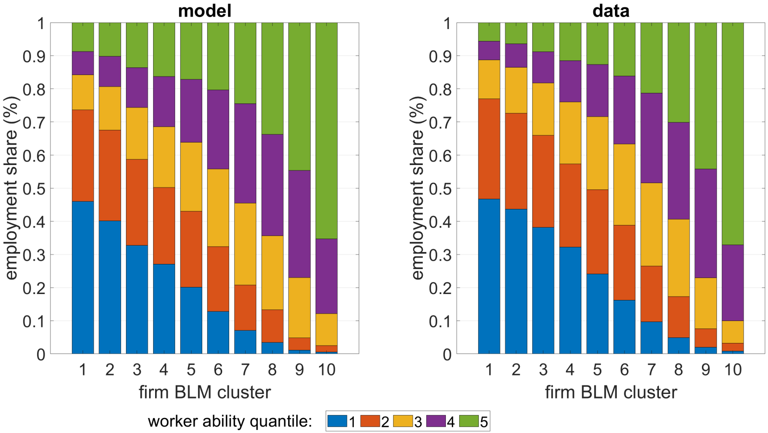
Network matching



back

Worker-firm sorting

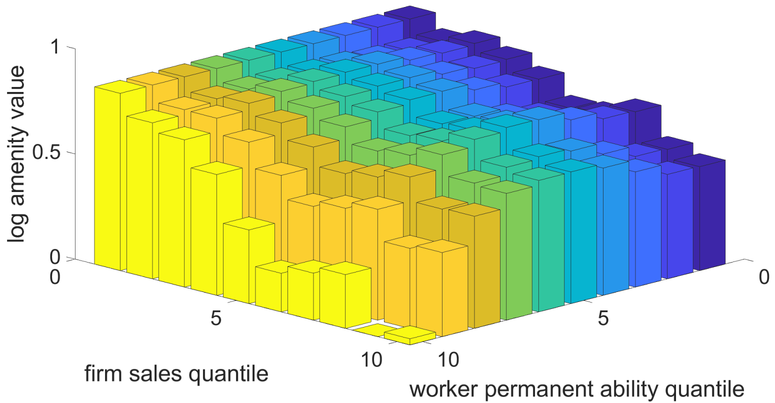
- Worker-firm sorting:



note: "BLM cluster" indicates k-means cluster of firm based on percentiles of within-firm earnings distribution

back

Amenities



back

A Shapley Approach for Counterfactual Simulations

- To illustrate, consider two sources of variation, Θ_A and Θ_B
- Suppose that an inequality outcome X such as earnings can be expressed as $X = \Theta_A + \Theta_B$, so that:

$$\text{var}\,(X) = \text{var}(\Theta_A) + \text{var}(\Theta_B) + 2\text{cov}(\Theta_A, \Theta_B)$$

A Shapley Approach for Counterfactual Simulations

- To illustrate, consider two sources of variation, Θ_A and Θ_B
- Suppose that an inequality outcome X such as earnings can be expressed as $X = \Theta_A + \Theta_B$, so that:

$$\text{var}(X) = \text{var}(\Theta_A) + \text{var}(\Theta_B) + 2\text{cov}(\Theta_A, \Theta_B)$$

- Change in $\text{var}(X)$ from eliminating heterogeneity in Θ_A: $\delta_{A1} = \text{var}(\Theta_A) + 2\text{cov}(\Theta_A, \Theta_B)$

A Shapley Approach for Counterfactual Simulations

- To illustrate, consider two sources of variation, Θ_A and Θ_B
- Suppose that an inequality outcome X such as earnings can be expressed as $X = \Theta_A + \Theta_B$, so that:

$$\text{var}\,(X) = \text{var}(\Theta_A) + \text{var}(\Theta_B) + 2\text{cov}(\Theta_A, \Theta_B)$$

- Change in $\text{var}\,(X)$ from eliminating heterogeneity in Θ_A: $\delta_{A1} = \text{var}\,(\Theta_A) + 2\text{cov}\,(\Theta_A, \Theta_B)$
- Change in $\text{var}\,(X)$ from eliminating heterogeneity in Θ_A without heterogeneity in Θ_B: $\delta_{A2} = \text{var}\,(\Theta_A)$

A Shapley Approach for Counterfactual Simulations

- To illustrate, consider two sources of variation, Θ_A and Θ_B
- Suppose that an inequality outcome X such as earnings can be expressed as $X = \Theta_A + \Theta_B$, so that:

$$\mathrm{var}\,(X) = \mathrm{var}(\Theta_A) + \mathrm{var}(\Theta_B) + 2\mathrm{cov}(\Theta_A, \Theta_B)$$

- Change in $\mathrm{var}\,(X)$ from eliminating heterogeneity in Θ_A: $\delta_{A1} = \mathrm{var}\,(\Theta_A) + 2\mathrm{cov}\,(\Theta_A, \Theta_B)$
- Change in $\mathrm{var}\,(X)$ from eliminating heterogeneity in Θ_A without heterogeneity in Θ_B: $\delta_{A2} = \mathrm{var}\,(\Theta_A)$
- Shapley contribution of Θ_A to $\mathrm{var}\,(X)$: $\frac{\delta_{A1}+\delta_{A2}}{2} = \mathrm{var}\,(\Theta_A) + \mathrm{cov}\,(\Theta_A, \Theta_B)$

A Shapley Approach for Counterfactual Simulations

- To illustrate, consider two sources of variation, Θ_A and Θ_B
- Suppose that an inequality outcome X such as earnings can be expressed as $X = \Theta_A + \Theta_B$, so that:

$$\text{var}(X) = \text{var}(\Theta_A) + \text{var}(\Theta_B) + 2\text{cov}(\Theta_A, \Theta_B)$$

- Change in $\text{var}(X)$ from eliminating heterogeneity in Θ_A: $\delta_{A1} = \text{var}(\Theta_A) + 2\text{cov}(\Theta_A, \Theta_B)$
- Change in $\text{var}(X)$ from eliminating heterogeneity in Θ_A without heterogeneity in Θ_B: $\delta_{A2} = \text{var}(\Theta_A)$
- Shapley contribution of Θ_A to $\text{var}(X)$: $\frac{\delta_{A1}+\delta_{A2}}{2} = \text{var}(\Theta_A) + \text{cov}(\Theta_A, \Theta_B)$
- Hence, Shapley approach is equivalent to splitting covariance equally between A and B in the linear case
  - but it generalizes to cases where outcomes cannot be expressed as a linear combination of primitives

A Shapley Approach for Counterfactual Simulations

- To illustrate, consider two sources of variation, Θ_A and Θ_B
- Suppose that an inequality outcome X such as earnings can be expressed as $X = \Theta_A + \Theta_B$, so that:

$$\text{var}(X) = \text{var}(\Theta_A) + \text{var}(\Theta_B) + 2\text{cov}(\Theta_A, \Theta_B)$$

- Change in var (X) from eliminating heterogeneity in Θ_A: $\delta_{A1} = \text{var}(\Theta_A) + 2\text{cov}(\Theta_A, \Theta_B)$
- Change in var (X) from eliminating heterogeneity in Θ_A without heterogeneity in Θ_B: $\delta_{A2} = \text{var}(\Theta_A)$
- Shapley contribution of Θ_A to var (X): $\frac{\delta_{A1}+\delta_{A2}}{2} = \text{var}(\Theta_A) + \text{cov}(\Theta_A, \Theta_B)$
- Hence, Shapley approach is equivalent to splitting covariance equally between A and B in the linear case
  - but it generalizes to cases where outcomes cannot be expressed as a linear combination of primitives
- For the production network, "eliminating heterogeneity" in customer/supplier matching means:
  - each firm matches with all buyers/sellers with equal probability
  - holding constant the total number of buyers/sellers for each firm
  - relationship productivity residuals $\tilde{\psi}_{ijt}$ set to mean of $\tilde{\psi}_{ijt}$ across all buyers/sellers of each firm

technical definition back

Technical Definition of the Shapley Approach

- Define the following
  - Θ: the estimated vector of values for all model primitives
  - $X(\Theta)$: the value of some equilibrium outcome X under Θ
  - $\{\theta_n\}_{n=1}^{N}$: some N covers of the parameter vector Θ, with $\mathcal{N} \equiv \{1, \cdots, N\}$

Technical Definition of the Shapley Approach

- Define the following
  - Θ: the estimated vector of values for all model primitives
  - $X(\Theta)$: the value of some equilibrium outcome X under Θ
  - $\{\theta_n\}_{n=1}^{N}$: some N covers of the parameter vector Θ, with $\mathcal{N} \equiv \{1, \cdots, N\}$
- Suppose we are interested in the value of X under some counterfactual parameters

Technical Definition of the Shapley Approach

- Define the following
  - Θ: the estimated vector of values for all model primitives
  - $X(\Theta)$: the value of some equilibrium outcome X under Θ
  - $\{\theta_n\}_{n=1}^{N}$: some N covers of the parameter vector Θ, with $\mathcal{N} \equiv \{1, \cdots, N\}$
- Suppose we are interested in the value of X under some counterfactual parameters
- Let $\hat{\Theta}_S \equiv \left\{\cup_{n\in S}\hat{\theta}_n\right\} \cup \{\cup_{n\notin S}\theta_n\}$ for some $S \subseteq \mathcal{N}$ denote the parameter vector with:
  - parameters in S set to counterfactual values $\hat{\theta}_n$
  - parameters not in S set to baseline values θ_n

Technical Definition of the Shapley Approach

- Define the following
  - Θ: the estimated vector of values for all model primitives
  - $X(\Theta)$: the value of some equilibrium outcome X under Θ
  - $\{\theta_n\}_{n=1}^{N}$: some N covers of the parameter vector Θ, with $\mathcal{N} \equiv \{1, \cdots, N\}$
- Suppose we are interested in the value of X under some counterfactual parameters
- Let $\hat{\Theta}_S \equiv \left\{\cup_{n\in S}\hat{\theta}_n\right\} \cup \{\cup_{n\notin S}\theta_n\}$ for some $S \subseteq \mathcal{N}$ denote the parameter vector with:
  - parameters in S set to counterfactual values $\hat{\theta}_n$
  - parameters not in S set to baseline values θ_n
- The Shapley value X_n for θ_n in relation to outcome X is:

$$X_n = \sum_{S\subseteq\mathcal{N}\setminus\{n\}} \frac{|S|!\,(N! - |S|! - 1)}{N!}\left[X\left(\hat{\Theta}_{S\cup\{n\}}\right) - X\left(\hat{\Theta}_S\right)\right]$$

back

Counterfactual Results with $\epsilon = 1$

- **Outcome**: variance of log firm earnings effect, $\text{var}(\log W_{it}) = 0.18$

	baseline	$\epsilon = 1$
supplier network	23.6	26.7
customer network	6.6	24.4
firm productivities	40.7	14.7
production complementarity	26.7	32.4
firm amenities	13.3	16.2
worker abilities	-10.8	-14.4

Each column shows the percentage of an inequality outcome accounted for by each source of variation.

back

Counterfactual Results with $\epsilon = 1$

- **Outcome**: covariance between firm earnings effects and firm size, $\text{cov}(\log W_{it}, \log R_{it}) = 0.57$

	baseline	$\epsilon = 1$
supplier network	35.3	39.3
customer network	15.8	35.9
firm productivities	44.8	17.1
production complementarity	8.0	13.2
firm amenities	1.7	2.3
worker abilities	-5.5	-7.7

Each column shows the percentage of an inequality outcome accounted for by each source of variation.

back